



**Intermediate Math Circles**  
**Wednesday November 05 2014**  
**Problem Set**

1. Jim calculates the average of a set of  $n$  numbers to be 10. Linlin removes the number 2 from the set and recalculates the average of the remaining numbers to be 14. What is the value of  $n$ ?
2. Mr. Math has more than 25 students in his class. He has more than 2 but fewer than 10 boys and more than 14 but fewer than 23 girls in his class. How many different class sizes would satisfy these conditions?
3. The five expressions  $2x + 1$ ,  $2x - 3$ ,  $x + 2$ ,  $x + 5$  and  $x - 3$  can be arranged in a different order so that the sum of the first three expressions is  $4x + 3$  and the sum of the last three expressions is  $4x + 4$ . What is the middle expression in the new list?
4. Solve  $x + 2 \leq 3x - 10$  and sketch your solution.
5. Solve  $10 + 7x < 4x + 9$  and sketch your solution.
6. Solve  $\frac{1}{2}(2 + 5x) \geq \frac{2}{3}(15 - 3x)$  and sketch your solution.
7. How many integer values of  $x$  satisfy  $\frac{x-1}{3} < \frac{5}{7} < \frac{x+4}{5}$ ?
8. How many positive integers  $p$  satisfy  $-1 < \sqrt{p} - \sqrt{121} < 1$ ?
9. If  $-4 < x < 6$  then determine  $a$  and  $b$  in  $a < 2x - 5 < b$ ?
10. What values of  $x$  satisfy the inequality  $-3 < 5 - \frac{2}{x} < 3$ ? Sketch your solution.
11. Solve  $2 - \frac{1}{x} < 3$  and sketch your solution.
12. Solve  $\frac{2}{x} + 3 \geq 4$  and sketch your solution.
13. If  $\frac{\left(\frac{a}{c} + \frac{a}{b} + 1\right)}{\left(\frac{b}{a} + \frac{b}{c} + 1\right)} = 11$  where  $a$ ,  $b$ , and  $c$  are positive integers, how many different ordered triples  $(a, b, c)$  are there such that  $a + 2b + c \leq 40$  is true?
14. Jane loves math problems and she receives 1200 new questions from her Math Teacher to complete. From past experience, she knows that:
  - Half of the problems will be difficult to solve,  $x$  minutes will be required for each
  - Two-thirds of the remainder are not as hard, she completes these in 40% of the time required for a difficult problem
  - The remaining questions are a snap, she only needs 75% of the time required for a difficult problem

Jane predicts that the total time required to complete all of the problems must be greater than or equal to 27300 minutes. What is the least amount of time she needs to spend on each difficult problem?