



Intermediate Math Circles
November 26, 2014
Some CIMC Solutions,
some cool questions,
and some Holiday Math Fun!

Here is the warmup problem to try as everyone arrives.

Find the sum of the following series.

$$22 + 23 + 24 + 25 + \cdots + 49 + 50$$

But my favourite Gauss story is....

But then Gauss went further.....



Find the sum of the natural numbers from 1 to 2014.

Evaluate $22 + 23 + 24 + 25 + \cdots + 49 + 50$

Find the sum of all powers of 5 from 5 to 2015.
i.e. Sum $5 + 10 + 15 + \cdots + 2010 + 2015$



Proof by Induction.

Step 1. First we show it works to start. i.e. Works for 1.

Step 2. We assume it works for $n = k$.

Then we have $1 + 2 + 3 + \cdots + (k - 1) + k = \frac{k(k+1)}{2}$

Step 3. Use the assumption to prove it works for $n = k + 1$.

i.e. $1 + 2 + 3 + \cdots + k + (k + 1) = \frac{(k+1)(k+2)}{2}$



Try these three.

1. $1 + 2 + 3 + \cdots + 149 + 150 + 200 + 201 + 202 + \cdots + 299 + 300$

2. $9 + 12 + 15 + \cdots + 99 + 102$

3. $4 + 7 + 10 + \cdots + 298 + 301$



A6. A positive integer is a prime number if it is greater than 1 and has no positive divisors other than 1 and itself.

The integer 43797 satisfies the following conditions:

- each pair of neighbouring digits (read from left to right) forms a two-digit prime number, and
- all of the prime numbers formed by these pairs are different.

What is the largest positive integer that satisfies both of these conditions?

Solution

Start with the two digit primes:

11,13,17,23,29,31,37,41,43,53,59,61,67,71,73,79,83,89,97

Split them into the not too useful primes with an even digit and the very useful totally odd primes.

Even: 23,29,41,43,61,67,83,89

Odd: 11,13,17,19,31,37,53,59,71,73,79,97

So something like 311371973 or 8971373119 would be good numbers.

In the odd group which digit is most common?

Which even goes on the front?

And the answer is.....



1a. Determine the average of the six integers 22,23,23,25,26,31.

1b. the average of the three numbers $y + 7$, $2y - 9$, $8y + 6$ is 27. What is the value of y ?

1c. Four positive integers, not necessarily different and each less than 100, have an average of 94. Determine, with explanation, the minimum possible value for one of these integers.



www.krazydad.com is a wonderful site for all sorts of Brain Math.
The following puzzles all come from this site.

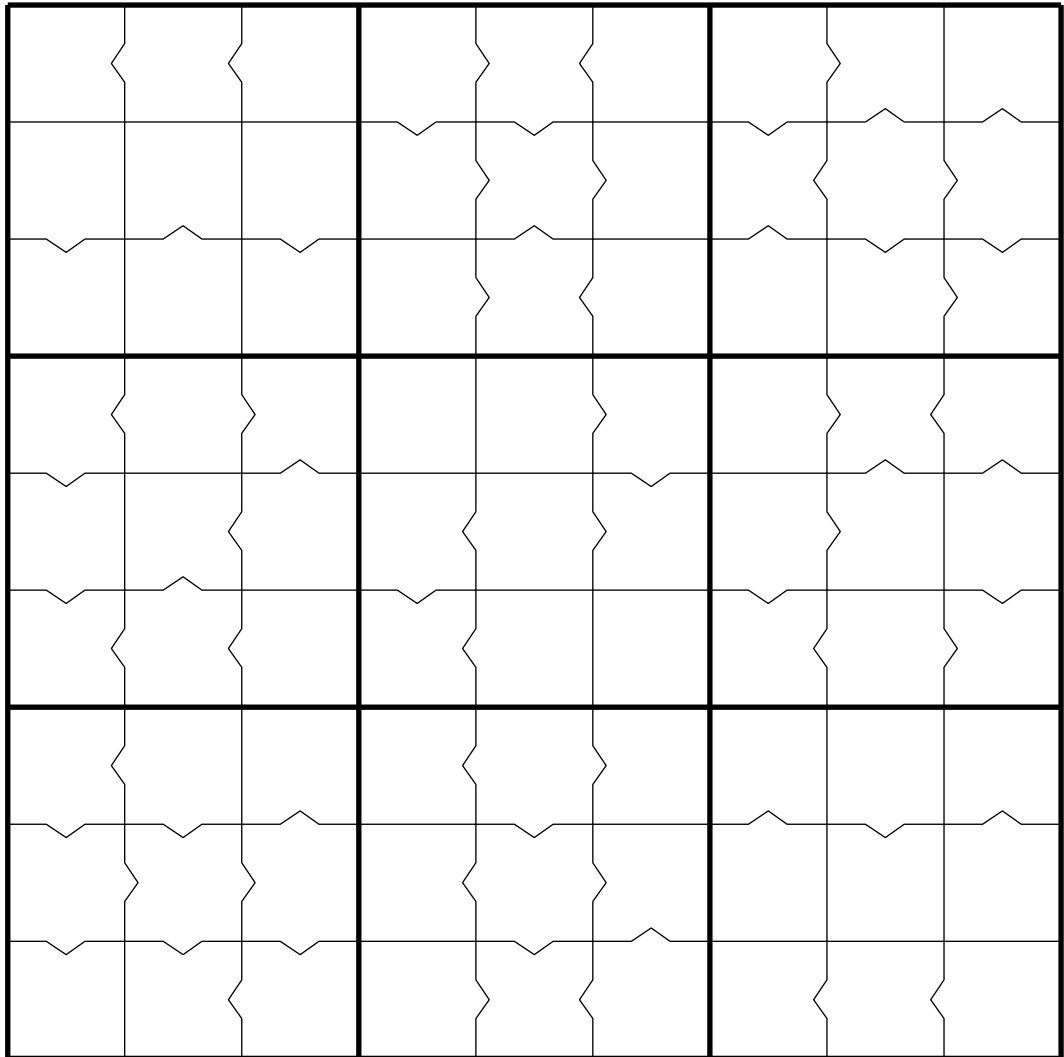
Sudoku #6

9			3			7		
4				7	1	8		
		3		5	2			9
2		4					1	
	8					2		5
5			6	2		1		
		7	1	4				2
		1			8			7

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Jigoku #8



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