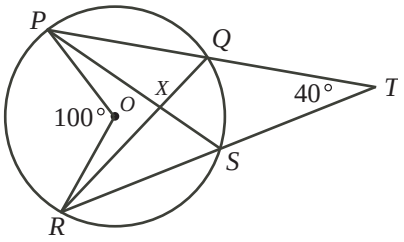
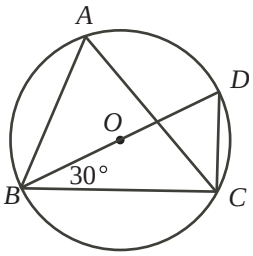
Intermediate Math Circles
Wednesday October 29 2014
Problem Set 4

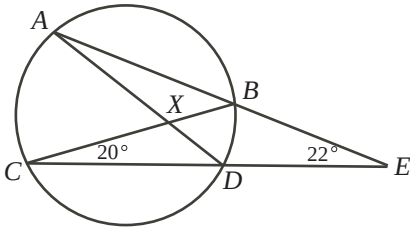
1. In the diagram, O is the centre of the circle. Determine the measure of $\angle QXS$.



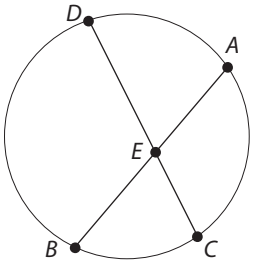
2. Determine the measure of $\angle BAC$.



3. Determine the measure of $\angle ADC$ and of $\angle AXB$.



4. AB and CD are two intersecting chords in a circle.

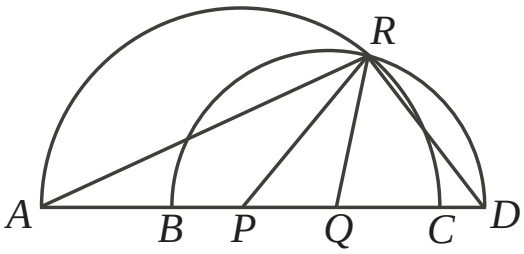


   a) If $AE = 6$, $BE = 4$ and $CE = 8$, determine the length of DE.

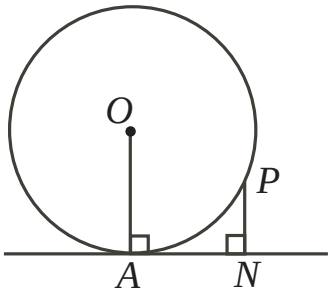
   b) If $AE = x$, $AB = 2x + 5$, $CE = x + 11$ and $CD = 2x + 7$, determine the value of x.

5. A *cyclic quadrilateral* is a quadrilateral that has all four of its vertices on the same circle. Prove that opposite angles are supplementary.

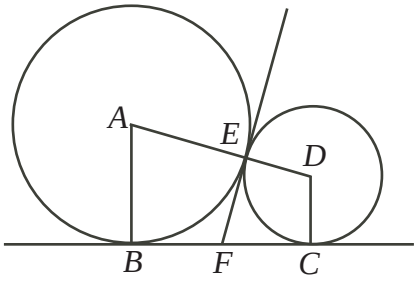
6. In the diagram, points B, P, Q, and C lie on line segment AD. The semi-circle with diameter AC has centre P and the semi-circle with diameter BD has centre Q. The two semi-circles intersect at R. If $\angle PRQ = 40°$, determine the measure of $\angle ARD$.



7. In the diagram, O is the centre of the circle, AN is tangent to the circle at A, P lies on the circle, and PN is perpendicular to AN. If $AN = 15$ and $PN = 9$, determine the radius of the circle.



8. In the diagram, a circle with centre A and radius 9 is tangent to a smaller circle with centre D and radius 4. Common tangents EF and BC are drawn to the circles making points of contact at E, B, and C. Determine the length of EF. (For this question you may have to use properties which make sense but are, as of yet, unproven.)



9. If O is the centre of the circle and $\angle BCD = 82°$, what is the value of x in degrees?