Grade 7/8 Math Circles
Fall 2014 - Nov. 25/26

Gauss Preparation Test

General Information
The Gauss contest is an opportunity for grade 7/8 students to have fun and challenge their mathematical problem solving skills.

Date and Registration

Registration Deadline: April 22, 2015
Test Writing Date: May 13, 2015

Format and Marking Scheme

- 60 minutes
- 25 multiple choice questions
- 150 marks:
  - Part A: 10 questions - 5 marks each
  - Part B: 10 questions - 6 marks each
  - Part C: 5 questions - 8 marks each
  - Unanswered Questions: 2 marks each (for up to 10 questions)

Contest Success Strategies

- **ELIMINATE** - choices that aren’t sensible answers, making it easier to guess
- **DRAW** - diagrams representing your scenario to help clear up misconceptions
- **MOVE ON** - from questions you are stuck on to get as many marks as possible
- **FOCUS** - on Part B and Part C questions as Part A shouldn’t pose a challenge
- **PRACTICE** - by studying from the contest bank on the CEMC website
- **LEARN** - techniques and short-cuts from past contest solutions
Mock Gauss Contest

Note that these are a combination of questions from the Grade 8 Gauss contests. Although the grades are listed for reference, all questions are applicable for both grades.

Part A - 5 marks each

1. Gauss Grade 8, 2007 (#1)

The value of \((2 \times 12) - (2 + 12)\)

\[(A) \ 34 \quad (B) \ 44 \quad (C) \ 10 \quad (D) \ -4 \quad (E) \ 32\]

\[(2 \times 12) - (2 + 12) = 24 - 14 = 10\]

2. Gauss Grade 8, 2010 (#8)

The time on a digital clock is 10:25. In minutes, what is the shortest length of time until all the digits on the clock will be equal to one another.

\[(A) \ 36 \quad (B) \ 107 \quad (C) \ 86 \quad (D) \ 46 \quad (E) \ 187\]

The next time on the clock when all digits are equal is 11:11, which will take 46 minutes.

3. Gauss Grade 8, 2006 (#3)

Jamie sells a camera for \(\$200.00\) and earns a commission rate of 25% on the sale. How much commission does he earn?

\[(A) \ \$25.00 \quad (B) \ \$50.00 \quad (C) \ \$250.00 \quad (D) \ \$75.00 \quad (E) \ \$100.00\]

\[0.25(\$200) = \frac{\$200}{4} = \$50\]

4. Gauss Grade 8, 2008 (#4)

The value of \((2 + 3)^2 - (2^2 + 1^2)\)

\[(A) \ 20 \quad (B) \ 6 \quad (C) \ 22 \quad (D) \ 8 \quad (E) \ 16\]

\[(2 + 3)^2 - (2^2 + 1^2) = 5^2 - 5 = 25 - 5 = 20\]
5. Gauss Grade 8, 2013 (#7)

Each letter of the English alphabet is written on a separate tile and placed in a bag. Alonso draws one letter at random from the bag. What is the probability that Alonso draws a letter that is in his name?

(A) \( \frac{1}{26} \)  (B) \( \frac{4}{26} \)  (C) \( \frac{5}{26} \)  (D) \( \frac{2}{26} \)  (E) \( \frac{3}{26} \)

There are 5 letters in Alonso that show up in the alphabet: a, l, o, n, s. Since there are 26 lets in the alphabet, the probability he draws a letter that is in his name is \( \frac{5}{26} \).

6. Gauss Grade 8, 2013 (#6)

What number goes in the box so that \( 10 \times 20 \times 30 \times 40 \times 50 = 100 \times 2 \times 300 \times 4 \times \Box \)?

(A) 0.5  (B) 5  (C) 50  (D) 500  (E) 5000

For the right side of the equation: \( 100 \times 2 \times 300 \times 4 = 10 \times 20 \times 30 \times 40 \times \Box \)

This tells us that 50 must go in the box to make the equation work.

7. Gauss Grade 8, 2011 (#9)

If \( x = 4 \) and \( y = x + 2 \) and \( z = 3y + -3 \), the value of \( z \) is?

(A) 8  (B) 21  (C) 30  (D) 15  (E) 12

If \( x = 4 \) then \( y = 4 + 2 = 6 \) and \( z = 3(6) + -3 = 18 - 3 = 15 \).

8. Gauss Grade 8, 2006 (#9)

In the diagram, \( AB \) and \( CD \) are straight lines.

The value of \( x \) is

(A) 50  (B) 60  (C) 70

(D) 130  (E) 230

Using supplementary angles, the other two angles of the triangles are \( 180^\circ - 60^\circ - 50^\circ = 70^\circ \) and \( 180^\circ - 120^\circ = 60^\circ \). Therefore, \( x = 180 - 70 - 60 = 50 \).
9. Gauss Grade 8, 2006 (#7)

The volume of a rectangular block is 120 cm$^3$. If the area of its base is 24 cm$^2$, what is its height?

(A) 5 cm  (B) 15 cm  (C) 0.2 cm  (D) 0.6 cm  (E) 1 cm

Let $h$ be the height of the rectangular block, in cm. Then if $Volume = 120$

\[ h \times 24 = 120 \]
\[ h = \frac{120}{24} \]
\[ h = 5 \]

10. Gauss Grade 8, 2007 (#3)

The graph shows the daily high and low temperatures last week in Waterloo. On which day of the week was the difference between the high and low temperatures the greatest?

(A) Monday  (B) Tuesday  
(C) Wednesday  (D) Thursday  
(E) Friday

We can see that the high and low temperatures differ the most Thursday.
Part B - 6 marks each

11. Gauss Grade 8, 2006 (#16)

In the diagram, what is the length of $BC$?

(A) 13  (B) 12  (C) 20

(D) 16  (E) 17

$\triangle ABD$ corresponds to the Pythagorean Triple of 3 4 5 which has been multiplied by 4. So we know the length of $BD$ is 12. For $BC$, call its length $c$. Then, using the Pythagorean Theorem we have

$$c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$$

12. Gauss Grade 8, 2012 (#14)

Half of the square root of a number is 1. The number is

(A) 2  (B) 4  (C) 8  (D) 9  (E) 16

Let $x$ be the number we are solving for. Then,

$$\frac{1}{2}\sqrt{x} = 1$$

$$\sqrt{x} = 2$$

$$x = 4$$

13. Gauss Grade 8, 2007 (#11)

Lily is 90 cm tall. If Anika is $\frac{4}{3}$ of the height of Lily, and Sadaf is $\frac{5}{4}$ of the height of Anika, how tall is Sadaf?

(A) 180 cm  (B) 70 cm  (C) 96 cm  (D) 120 cm  (E) 150 cm

Anika is $\frac{4}{3} \times 90\text{cm} = 120\text{cm}$ tall. Therefore, Sadaf is $\frac{5}{4} \times 120\text{cm} = 150\text{cm}$ tall.
14. Gauss Grade 8, 2007 (#18)

The number \( n \) is doubled and then has \( y \) added to it. The result is then divided by 2 and has the original number \( n \) subtracted from it. The final result is

\[
\frac{2n + y}{2} - n = \frac{2n + y - 2n}{2} = \frac{y}{2}
\]

(A) \( n \)  \hspace{1cm} (B) \( y \)  \hspace{1cm} (C) \( n + y \)  \hspace{1cm} (D) \( \frac{n + y}{2} \)  \hspace{1cm} (E) \( \frac{y}{2} \)

15. Gauss Grade 8, 2007 (#20)

Lori took a 240 km trip to Waterloo. On her way there, her average speed was 120km/h. She was stopped for speeding, so on her way home her average speed was 80km/h. What was her average speed, in km/h, for the entire round-trip?

(A) 90  \hspace{1cm} (B) 96  \hspace{1cm} (C) 108  \hspace{1cm} (D) 102  \hspace{1cm} (E) 110

It took Lori 2 hours to travel to Waterloo (she travelled 240 km at 120km/h), and 3 hours to travel back (she travelled another 240 km at 80km/h). Also, she travelled for a total distance of 480 km, since one way there was 240 km. Since she spend 5 hours travelling in total, her average speed, in km/h, for the entire round-trip was \( \frac{480}{5} = 96 \).

16. Gauss Grade 8, 2008 (#17)

The decimal expansion of \( \frac{2}{13} \) is the repeating decimal 0.153846. What digit occurs in the 64th place after the decimal point? \( \frac{2}{13} = 0.153846153846153846153846... \)

(A) 8  \hspace{1cm} (B) 6  \hspace{1cm} (C) 5  \hspace{1cm} (D) 4  \hspace{1cm} (E) 3

The digits after the decimal form a repeating pattern in blocks of 6. So we take \( \frac{64}{6} \), which gives us a quotient of 10. So, after the pattern has repeated 10 times, we are at the \( 6 \times 10 = 60 \)th digit in the decimal. Then we just move 4 more decimals over to get to the 64th decimal place and we land on 8.
17. **Gauss Grade 8, 2010 (#14)**

Gina plays 5 games as a hockey goalie. The table shows the number of shots on her net and her saves for each game. What percentage of the total shots did she save?

<table>
<thead>
<tr>
<th>Game</th>
<th>Shots</th>
<th>Saves</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>21</td>
</tr>
</tbody>
</table>

(A) 52  (B) 65  (C) 80

(D) 82  (E) 85

Total shots = 10 + 13 + 7 + 11 + 24 = 65
Total saves = 7 + 9 + 6 + 9 + 21 = 52

\[
\frac{52}{65} = 80\%
\]

18. **Gauss Grade 8, 2013 (#17)**

**PQRS** is a rectangle with diagonals **PR** and **QS**, as shown. The value of \(y\) is

(A) 30  (B) 40  (C) 45

(D) 50  (E) 60

Since **PQRS** is a rectangle, we know that \(\angle PQR = 90^\circ\). So,

\[
4x + 5x = 90 \\
9x = 90 \\
x = 10
\]

Then, by alternate angles

\[
y = 5x \\
y = 50
\]
19. **Gauss Grade 8, 2012 (#13)**

Three numbers have a mean (average) of 9. The mode of these three numbers is 12. What is the smallest of these three numbers.

(A) 1  (B) 2  (C) 3  (D) 4  (E) 5

Since the mode of these three numbers is 12, we know 12 must appear twice. Also, if $x$ is our third number, then

\[
\frac{12 + 12 + x}{3} = 9
\]

\[
\frac{24 + x}{3} = 27
\]

\[
x = 3
\]

20. **Gauss Grade 8, 2010 (#19)**

In the sequence shown, each figure after the first is formed by adding 4 squares to the previous figure. How many squares form Figure 2010?

(A) 8037  (B) 8040  (C) 8043  

(D) 6030  (E) 6026

This is an arithmetic sequence. In particular

\[a_n = 1 + 4(n - 1)\]

So we plug in 2010 for $n$ in the equation above to get

\[a_{2010} = 1 + 4(2009) = 8037\]
Part C - 8 marks each

21. Gauss Grade 8, 2010 (#20)

In \( \triangle PQR \), a line segment is drawn from \( P \) to point \( S \) on side \( QR \). If \( \triangle PQS \) and \( \triangle PRS \) have the same area, which of the following statements must be true?

(A) \( PQ = PR \)  \hfill (B) \( PS = PQ \)  \hfill (C) \( QR = PS \)

(D) \( QS = SR \)  \hfill (E) \( PQ = QR \)

Note that the heights of both triangles are the same. Let \( h \) be this height and since the area of the two triangles are equal we have

\[
\frac{QS \times h}{2} = \frac{SR \times h}{2}
\]

\[
QS \times h = SR \times h
\]

\[
QS = SR
\]

22. Gauss Grade 8, 2010 (#22)

The values \( r, s, t, \) and \( u \) are 2, 3, 4, 5, but not necessarily in that order. What is the largest possible value of \( r \times s + u \times r + t \times r \)?

(A) 24 \hfill (B) 45 \hfill (C) 33 \hfill (D) 40 \hfill (E) 49

We let \( r = 5 \) since \( r \) gets multiplied 3 times, more than any other value, in this expression. Since \( r \) gets multiplied by the other three values and we are finding the sum of these 3 multiplications, it does not matter what we let the other variables equal. So we have

\[
5 \times 2 + 3 \times 5 + 4 \times 5 = 45
\]
23. Gauss Grade 8, 2006 (#24)

In the diagram, the grid has 150 rows and 150 columns, numbered from 1 to 150. In row 1, every box is shaded. In row 2, every second box is shaded. In row 3, every third box is shaded. The shading continues this way, so that every $n$th box in row $n$ is shaded. Which column has the greatest number of shaded boxes?

(A) 20    (B) 36    (C) 64

(D) 85    (E) 88

The number of shaded boxes in each respective column corresponds to the number of factors the value of the column has. So out of our 5 possible answers, we list all of their factors and choose the one with the most.

20: 1,2,4,5,10,20

36: 1,2,3,4,6,9,12,18,36

64: 1,2,4,8,16,32,64

85: 1,5,17,85

88: 1,2,4,8,11,22,44,88

Since 36 has the most factors, the 36th column will have the greatest number of shaded boxes.
24. **Gauss Grade 8, 2007 (#21)** In the diagram, \(ABCD\) is a square with side length 6, and \(WXYZ\) is a rectangle with \(ZY = 10\) and \(XY = 6\). Also, \(AD\) and \(WX\) are perpendicular. If the shaded area is equal to half of the area of \(WXYZ\), the length of \(AP\) is

(A) 1  (B) 1.5  (C) 4  
(D) 2  (E) 2.5

Firstly, the area of the shaded region is 30 (half of the area of rectangle \(WXYZ\)). Since the area of the square in the middle is \(6 \times 6 = 36\) and the shaded region is a rectangle, we know that the area of the square is also equal to \(30 + 6 \times \text{AP}\). Now we solve for \(AP\):

\[
30 + 6 \times \text{AP} = 36 \\
6 \times \text{AP} = 6 \\
\text{AP} = 1
\]

25. **Gauss Grade 8, 2010 (#21)**

In the diagram, \(AB\) is parallel to \(DC\) and \(ACE\) is a straight line. The value of \(x\) is

(A) 35  (B) 30  (C) 40  
(D) 45  (E) 50

Since \(\angle ACE\) is a straight angle, \(\angle ACB = 108^\circ - 105^\circ = 75^\circ\).
In \(\triangle ABC\), \(\angle BAC = 180^\circ - \angle ABC - \angle ACB = 180^\circ - 75^\circ - 75^\circ = 30^\circ\).
Since \(AB\) is parallel to \(DC\), \(\angle ACD = \angle BAC = 30^\circ\) (alternate angles).
In \(\triangle ADC\), \(\angle DAC = 180^\circ - \angle ADC - \angle ACD = 180^\circ - 115^\circ - 30^\circ = 35^\circ\).
Thus, the value of \(x\) is 35°.