Faculty of Mathematics
Waterloo, Ontario

Centre for Education in
Mathematics and Computing

# Grade 7/8 Math Circles

Fall 2014 - Nov. 25/26

***Gauss Preparation Test***

## General Information

The Gauss contest is an opportunity for grade 7/8 students to have fun and challenge their mathematical problem solving skills

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### Date and Registration

**Registration Deadline**: April 22, 2015

**Test Writing Date**: May 13, 2015

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### Format and Marking Scheme

- 60 minutes
- 25 multiple choice questions
- 150 marks:
  - Part A: 10 questions - 5 marks each
  - Part B: 10 questions - 6 marks each
  - Part C: 5 questions - 8 marks each
  - Unanswered Questions: 2 marks each (for up to 10 questions)

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### Contest Success Strategies

- **ELIMINATE** - choices that aren't sensible answers, making it easier to guess
- **DRAW** - diagrams representing your scenario to help clear up misconceptions
- **MOVE ON** - from questions you are stuck on to get as many marks as possible
- **FOCUS** - on Part B and Part C questions as Part A shouldn't pose a challenge
- **PRACTICE** - by studying from the contest bank on the CEMC website
- **LEARN** - techniques and short-cuts from past contest solutions

# Mock Gauss Contest

*Note that these are a combination of questions from the Grade 8 Gauss contests. Although the grades are listed for reference, all questions are applicable for both grades.*

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## Part A - 5 marks each

1. **Gauss Grade 8, 2007 (#1)**

   The value of $(2 \times 12) - (2 + 12)$

   **(A)** 34 **(B)** 44 **(C)** 10 **(D)** -4 **(E)** 32

   $(2 \times 12) - (2 + 12) = 24 - 14 = 10$

2. **Gauss Grade 8, 2010 (#8)**

   The time on a digital clock is 10:25. In minutes, what is the shortest length of time until all the digits on the clock will be equal to one another.

   **(A)** 36 **(B)** 107 **(C)** 86 **(D)** 46 **(E)** 187

   The next time on the clock when all digits are equal is 11:11, which will take 46 minutes.

3. **Gauss Grade 8, 2006 (#3)**

   Jamie sells a camera for \$200.00 and earns a commission rate of 25% on the sale. How much commission does he earn?

   **(A)** \$25.00 **(B)** \$50.00 **(C)** \$250.00 **(D)** \$75.00 **(E)** \$100.00

   $0.25(\$200) = \dfrac{\$200}{4} = \$50$

4. **Gauss Grade 8, 2008 (#4)**

   The value of $(2 + 3)^2 - (2^2 + 1^2)$

   **(A)** 20 **(B)** 6 **(C)** 22 **(D)** 8 **(E)** 16

   $(2 + 3)^2 - (2^2 + 1^2) = 5^2 - 5 = 25 - 5 = 20$

5. **Gauss Grade 8, 2013 (#7)**

Each letter of the English alphabet is written on a separate tile and placed in a bag. Alonso draws one letter at random from the bag. What is the probability that Alonso draws a letter that is in his name?

**(A)** $\frac{1}{26}$ **(B)** $\frac{4}{26}$ **(C)** $\frac{5}{26}$ **(D)** $\frac{2}{26}$ **(E)** $\frac{3}{26}$

There are 5 letters in Alonso that show up in the alphabet: a, l, o, n, s. Since there are 26 lets in the alphabet, the probability he draws a letter that is in his name is $\frac{5}{26}$

6. **Gauss Grade 8, 2013 (#6)**

What number goes in the box so that $10 \times 20 \times 30 \times 40 \times 50 = 100 \times 2 \times 300 \times 4 \times \square$?

**(A)** 0.5 **(B)** 5 **(C)** 50 **(D)** 500 **(E)** 5000

For the right side of the equation: $100 \times 2 \times 300 \times 4 = 10 \times 20 \times 30 \times 40 \times \square$

This tell us that 50 must go in the box to make the equation work.

7. **Gauss Grade 8, 2011 (#9)**

If $x = 4$ and $y = x + 2$ and $z = 3y + -3$, the value of $z$ is?

**(A)** 8 **(B)** 21 **(C)** 30 **(D)** 15 **(E)** 12

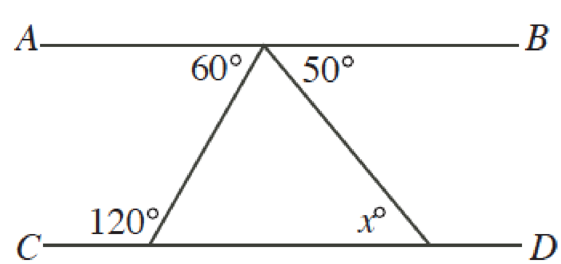
If $x = 4$ then $y = 4 + 2 = 6$ and $z = 3(6) + -3 = 18 - 3 = 15$

8. **Gauss Grade 8, 2006 (#9)**

In the diagram, $AB$ and $CD$ are straight lines. The value of $x$ is

**(A)** 50 **(B)** 60 **(C)** 70

**(D)** 130 **(E)** 230




Using supplementary angles, the other two angles of the triangles are $180° - 60° - 50° = 70°$ and $180° - 120° = 60°$. Therefore, $x = 180 - 70 - 60 = 50$.

9. **Gauss Grade 8, 2006 (#7)**

The volume of a rectangular block is 120 $\text{cm}^3$. If the area of its base is 24 $\text{cm}^2$, what is its height?

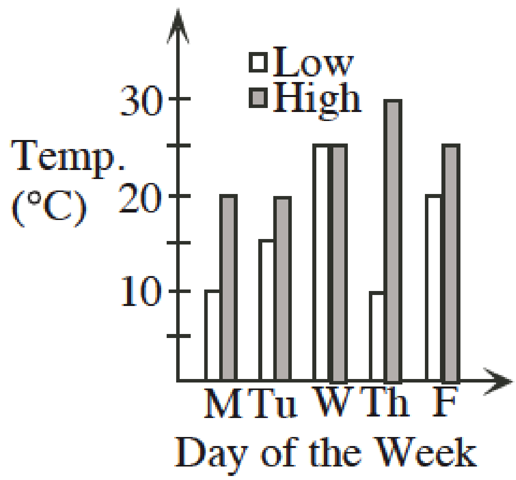
**(A)** 5 cm **(B)** 15 cm **(C)** 0.2 cm **(D)** 0.6 cm **(E)** 1 cm

Let $h$ be the height of the rectangular block, in cm. Then if $Volume = 120$

$$\begin{aligned} h \times 24 &= 120 \\ h &= \frac{120}{24} \\ h &= 5 \end{aligned}$$

10. **Gauss Grade 8, 2007 (#3)**

The graph shows the daily high and low temperatures last week in Waterloo. On which day of the week was the difference between the high and low temperatures the greatest?




**(A)** Monday **(B)** Tuesday

**(C)** Wednesday **(D)** Thursday

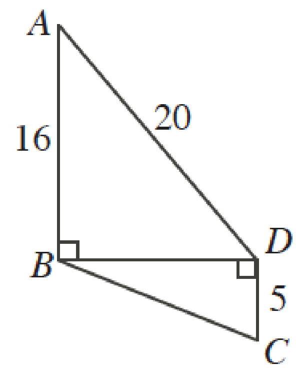
**(E)** Friday

We can see that the high and low temperatures differ the most Thursday.

## Part B - 6 marks each

11. **Gauss Grade 8, 2006 (#16)**

    In the diagram, what is the length of $BC$?

    **(A)** 13 **(B)** 12 **(C)** 20

    **(D)** 16 **(E)** 17

    $\triangle ABD$ corresponds to the Pythagorean Triple of 3 4 5 which has been multiplied by 4. So we know the length of $BD$ is 12. For $BC$, call its length $c$. Then, using the Pythagorean Theorem we have

    $$c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$$

12. **Gauss Grade 8, 2012 (#14)**

    Half of the square root of a number is 1. The number is

    **(A)** 2 **(B)** 4 **(C)** 8 **(D)** 9 **(E)** 16

    Let $x$ be the number we are solving for. Then,

    $$\frac{1}{2}\sqrt{x} = 1$$
    $$\sqrt{x} = 2$$
    $$x = 4$$

13. **Gauss Grade 8, 2007 (#11)**

    Lily is 90 cm tall. If Anika is $\frac{4}{3}$ of the height of Lily, and Sadaf is $\frac{5}{4}$ of the height of Anika, how tall is Sadaf?

    **(A)** 180 cm **(B)** 70 cm **(C)** 96 cm **(D)** 120 cm **(E)** 150 cm

    Anika is $\frac{4}{3} \times 90cm = 120cm$ tall. Therefore, Sadaf is $\frac{5}{4} \times 120cm = 150cm$ tall.

14. **Gauss Grade 8, 2007 (#18)**

The number $n$ is doubled and then has $y$ added to it. The result is then divided by 2 and has the original number $n$ subtracted from it. The final result is

**(A)** $n$ **(B)** $y$ **(C)** $n+y$ **(D)** $\frac{n+y}{2}$ **(E)** $\frac{y}{2}$

$$\begin{aligned} & \frac{2n+y}{2} - n \\ = & \frac{2n+y-2n}{2} \\ = & \frac{y}{2} \end{aligned}$$

15. **Gauss Grade 8, 2007 (#20)**

Lori took a 240 km trip to Waterloo. On her way there, her average speed was 120km/h. She was stopped for speeding, so on her way home her average speed was 80km/h. What was her average speed, in km/h, for the entire round-trip?

**(A)** 90 **(B)** 96 **(C)** 108 **(D)** 102 **(E)** 110

It took Lori 2 hours to travel to Waterloo (she travelled 240 km at 120km/h), and 3 hours to travel back (she travelled another 240 km at 80km/h). Also, she travelled for a total distance of 480 km, since one way there was 240 km. Since she spend 5 hours travelling in total, her average speed, in km/h, for the entire round-trip was $\frac{480}{5} = 96$.

16. **Gauss Grade 8, 2008 (#17)**

The decimal expansion of $\frac{2}{13}$ is the repeating decimal $0.\overline{153846}$. What digit occurs in the 64th place after the decimal point? ($\frac{2}{13} = 0.153846153846153846153846...$)

**(A)** 8 **(B)** 6 **(C)** 5 **(D)** 4 **(E)** 3

The digits after the decimal form a repeating pattern in blocks of 6. So we take $\frac{64}{6}$, which gives us a quotient of 10. So, after the pattern has repeated 10 times, we are at the $6 \times 10 = 60$th digit in the decimal. Then we just move 4 more decimals over to get to the 64th decimal place and we land on 8.

17. **Gauss Grade 8, 2010 (#14)**

Gina plays 5 games as a hockey goalie. The table shows the number of shots on her net and her saves for each game. What percentage of the total shots did she save?

| Game | Shots | Saves |
|---|---|---|
| 1 | 10 | 7 |
| 2 | 13 | 9 |
| 3 | 7 | 6 |
| 4 | 11 | 9 |
| 5 | 24 | 21 |

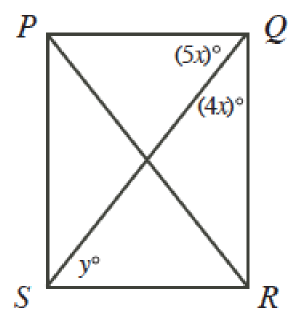
**(A)** 52 **(B)** 65 **(C)** 80 **(D)** 82 **(E)** 85

Total shots $= 10 + 13 + 7 + 11 + 24 = 65$
Total saves $= 7 + 9 + 6 + 9 + 21 = 52$

$\frac{52}{65} = 80\%$

18. **Gauss Grade 8, 2013 (#17)**

$PQRS$ is a rectangle with diagonals $PR$ and $QS$, as shown. The value of $y$ is




**(A)** 30 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 60

Since $PQRS$ is a rectangle, we know that $\angle PQR = 90°$. So,

$$\begin{aligned} 4x + 5x &= 90 \\ 9x &= 90 \\ x &= 10 \end{aligned}$$

Then, by alternate angles

$$\begin{aligned} y &= 5x \\ y &= 50 \end{aligned}$$

19. **Gauss Grade 8, 2012 (#13)**

Three numbers have a mean (average) of 9. The mode of these three numbers is 12. What is the smallest of these three numbers.

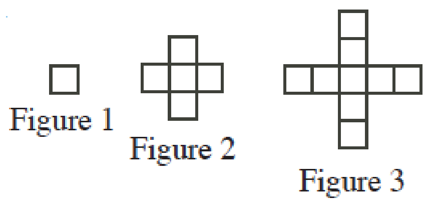
**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Since the mode of these three numbers is 12, we know 12 must appear twice. Also, if $x$ is our third number, then

$$\begin{aligned} \frac{12+12+x}{3} &= 9 \\ 24+x &= 27 \\ x &= 3 \end{aligned}$$

20. **Gauss Grade 8, 2010 (#19)**

In the sequence shown, each figure after the first is formed by adding 4 squares to the previous figure. How many squares form Figure 2010?




**(A)** 8037 **(B)** 8040 **(C)** 8043

**(D)** 6030 **(E)** 6026

This is an arithmetic sequence. In particular

$$a_n = 1 + 4(n-1)$$

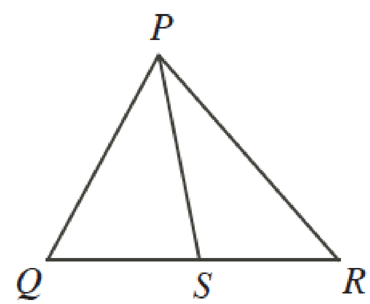
So we plug in 2010 for $n$ in the equation above to get

$$a_{2010} = 1 + 4(2009) = 8037$$

## Part C - 8 marks each

21. **Gauss Grade 8, 2010 (#20)**

In $\triangle PQR$, a line segment is drawn from $P$ to point $S$ on side $QR$. If $\triangle PQS$ and $\triangle PRS$ have the same area, which of the following statements *must* be true?




**(A)** $PQ = PR$ **(B)** $PS = PQ$ **(C)** $QR = PS$

**(D)** $QS = SR$ **(E)** $PQ = QR$

Note that the heights of both triangles are the same. Let $h$ be this height and since the area of the two triangles are equal we have

$$\begin{aligned} \frac{QS \times h}{2} &= \frac{SR \times h}{2} \\ QS \times h &= SR * h \\ QS &= SR \end{aligned}$$

22. **Gauss Grade 8, 2010 (#22)**

The values $r$, $s$, $t$, and $u$ are 2, 3, 4, 5, but not necessarily in that order. What is the largest possible value of $r \times s + u \times r + t \times r$?

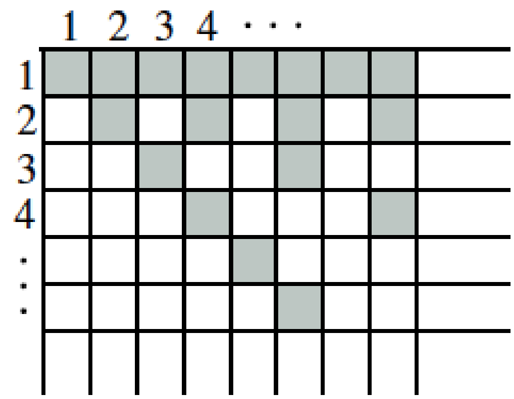
**(A)** 24 **(B)** 45 **(C)** 33 **(D)** 40 **(E)** 49

We let $r = 5$ since $r$ gets multiplied 3 times, more than any other value, in this expression. Since $r$ gets multiplied by the other three values and we are finding the sum of these 3 multiplications, it does not matter what we let the other variables equal. So we have

$$5 \times 2 + 3 \times 5 + 4 \times 5 = 45$$

23. **Gauss Grade 8, 2006 (#24)**

In the diagram, the grid has 150 rows and 150 columns, numbered from 1 to 150. In row 1, every box is shaded. In row 2, every second box is shaded. In row 3, every third box is shaded. The shading continues this way, so that every $n$th box in row $n$ is shaded. Which *column* has the greatest number of shaded boxes?




**(A)** 20 **(B)** 36 **(C)** 64

**(D)** 85 **(E)** 88

The number of shaded boxes in each respective column corresponds to the number of factors the value of the column has. So out of our 5 possible answers, we list all of their factors and choose the one with the most.

20: 1,2,4,5,10,20

36: 1,2,3,4,6,9,12,18,36

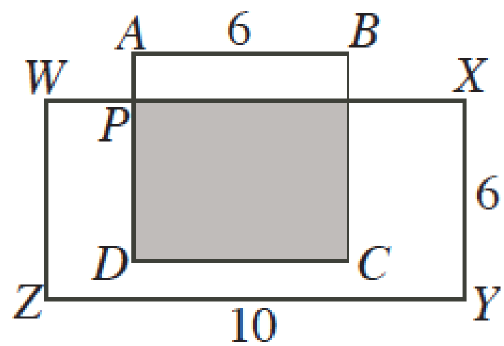
64: 1,2,4,8,16,32,64

85: 1,5,17,85

88: 1,2,4,8,11,22,44,88

Since 36 has the most factors, the 36th column will have the greatest number of shaded boxes.

24. **Gauss Grade 8, 2007 (#21)** In the diagram, $ABCD$ is a square with side length 6, and $WXYZ$ is a rectangle with $ZY = 10$ and $XY = 6$. Also, $AD$ and $WX$ are perpendicular. If the shaded area is equal to half of the area of $WXYZ$, the length of $AP$ is




**(A)** 1 **(B)** 1.5 **(C)** 4

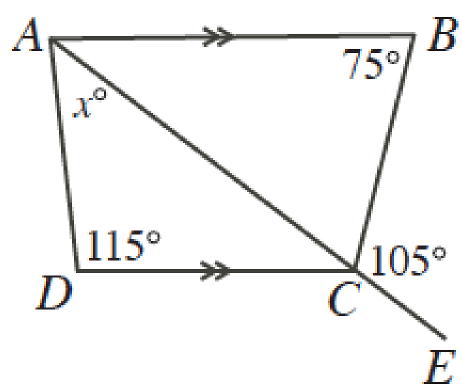
**(D)** 2 **(E)** 2.5

Firstly, the area of the shaded region is 30 (half of the area of rectangle WXYZ). Since the area of the square in the middle is $6 \times 6 = 36$ and the shaded region is a rectangle, we know that the area of the square is also equal to $30 + 6 \times AP$. Now we solve for $AP$:

$$\begin{aligned} 30 + 6 \times AP &= 36 \\ 6 \times AP &= 6 \\ AP &= 1 \end{aligned}$$

25. **Gauss Grade 8, 2010 (#21)**

In the diagram, $AB$ is parallel to $DC$ and $ACE$ is a straight line. The value of $x$ is




**(A)** 35 **(B)** 30 **(C)** 40

**(D)** 45 **(E)** 50

Since $\angle ACE$ is a straight angle, $\angle ACB = 108° - 105° = 75°$.
In $\triangle ABC$, $\angle BAC = 180° - \angle ABC - \angle ACB = 180° - 75° - 75° = 30°$.
Since $AB$ is parallel to DC, $\angle ACD = \angle BAC = 30°$ (alternate angles).
In $\triangle ADC$, $\angle DAC = 180° - \angle ADC - \angle ACD = 180° - 115° - 30° = 35°$.
Thus, the value of $x$ is 35°.