UNIVERSITY OF WATERLOO

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Centre for Education in
Mathematics and Computing

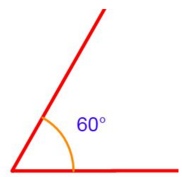
Grade 7/8 Math Circles

October 14/15, 2014

Angles

Angles

An **angle** is the space between two lines that intersect each other.



Terms and angles you should know:

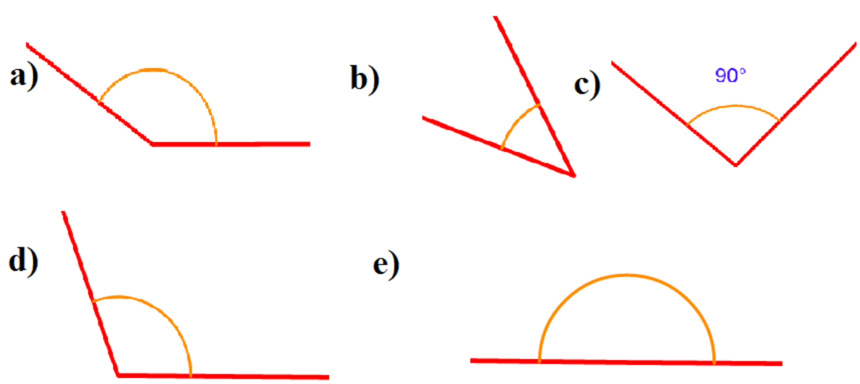
An **acute** angle is an angle less then 90°.

A **right** angle is a 90°angle.

An **obtuse** angle is more than 90°but less then 180°

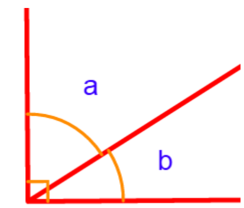
A **straight** angle is a 180°angle.

Classify these angles as on of the above:

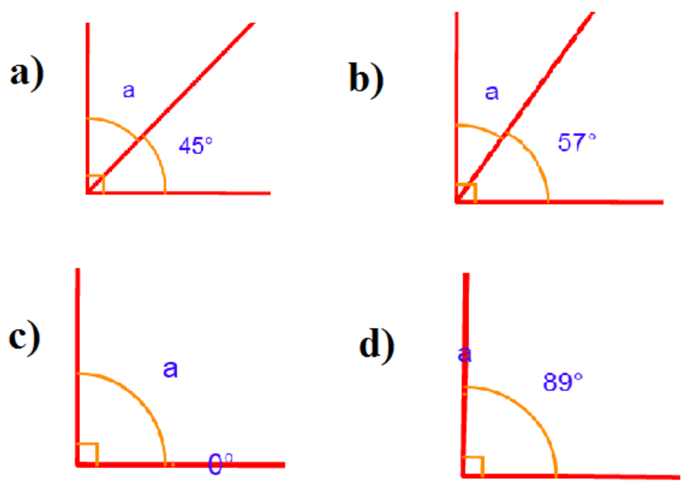


a) b) c) d) e)

Two angles are **complementary** if they add up to 90°. Below, a and b are complementary since $a + b = 90$

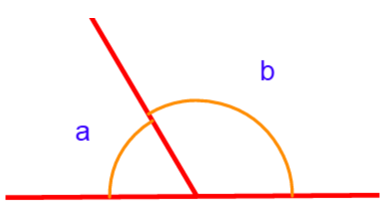


Find the complementary angle for the given angles(find a):

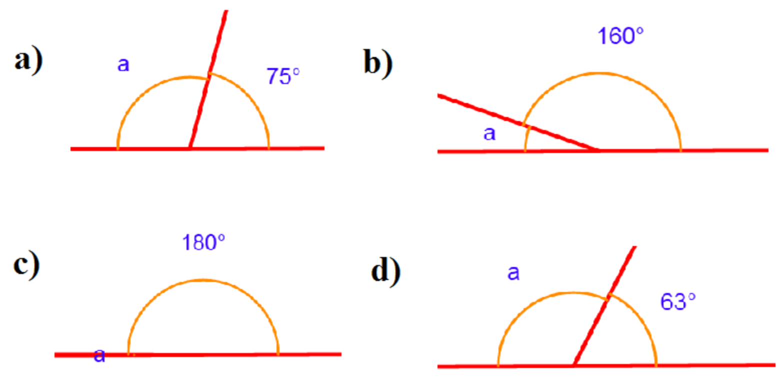


a) b) c) d)

Two angles are **supplementary**if they add up to 180°. Below, a and b are supplementary since $a + b = 180$



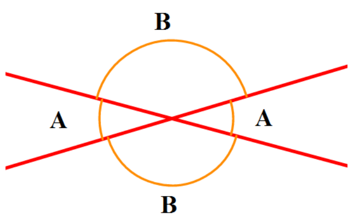
Find the supplementary angle for the given angles(find a):



a) b) c) d)

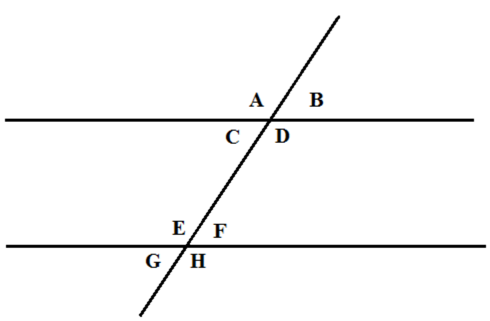
Trick to remember: C comes before S and 90 comes before 180

An **opposite** angle is the angle that is directly opposite from a given angle when two lines intersect.

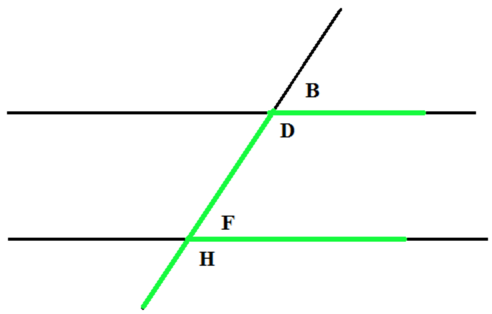


Can we explain this using our knowledge of supplementary angles?

A **transversal** is a line that intersects two or more lines. If these lines are parallel, the angles around these lines have very nice properties.

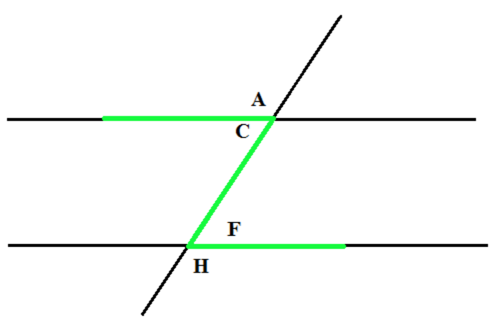


Corresponding angles are angles that are on the same side of the traversal as well as on the same side of each parallel line. Corresponding angles are equal to each other. We sometimes refer to these as F angles.



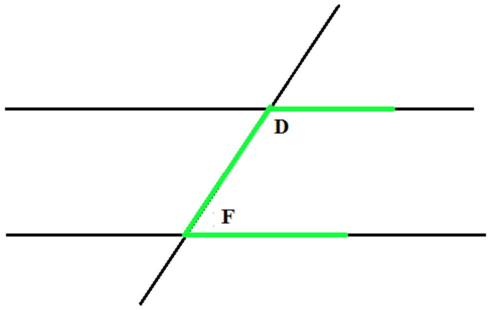
In the picture above there are 2 pairs of corresponding angles: D=H and B=F.

Alternate angles are angles that are on opposite sides of the transversal and on opposite sides of the parallel lines. Alternate angles are equal to each other. We sometimes refer to these as Z angles.



In the picture above there are 2 pairs of corresponding angles: C=F and A=H.

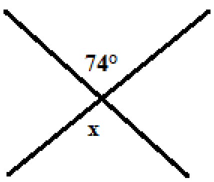
Interior angles are angles on the same side of a transversal and on opposite sides of the parallel lines. These angles add up to 180°. These are sometimes referred to as C angles.



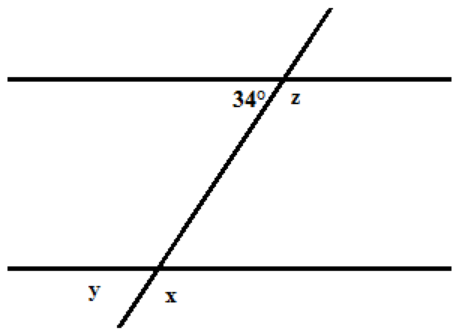
In the picture above D and F are interior angles. $D + F = 180$

Find the missing angles.

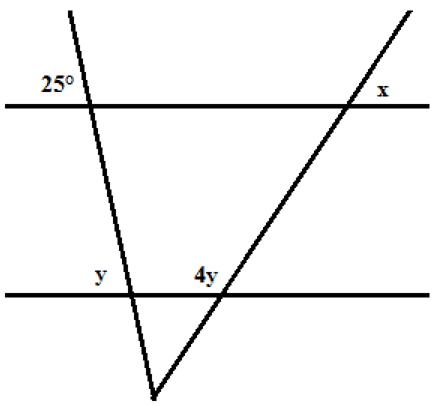
a)



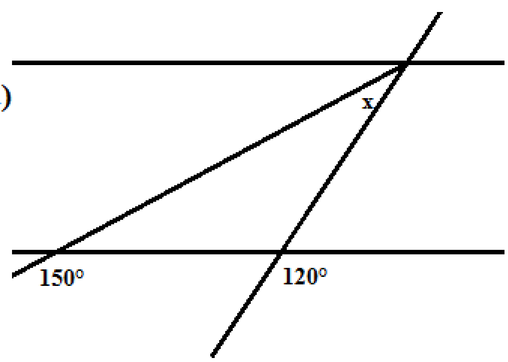
b)



c)



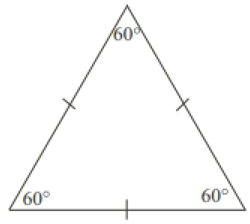
d)



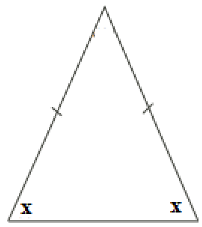
Triangles and More

First, lets talk about the 3 types of triangles.

Equilateral: These triangles have three equal length sides and three equal angles.

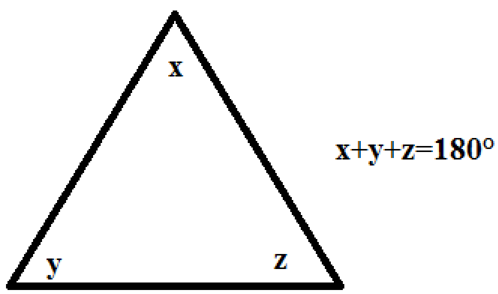


Isosceles: These triangles have two equal length sides and two equal angles.



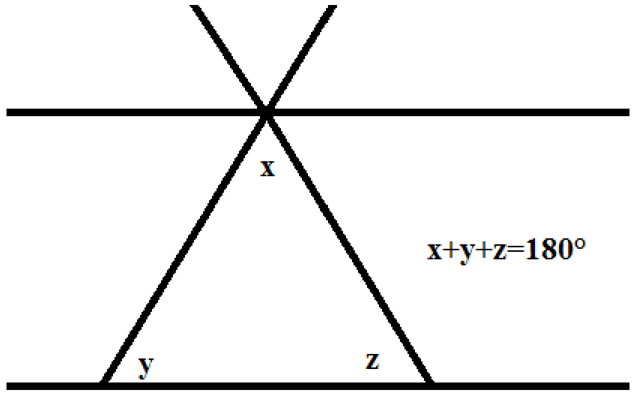
Scalene: No sides are equal and no angles are equal.

What is the sum of the interior angles of a triangle? 180. Any triangle, regardless of size has 3 interior angles that add up to 180.

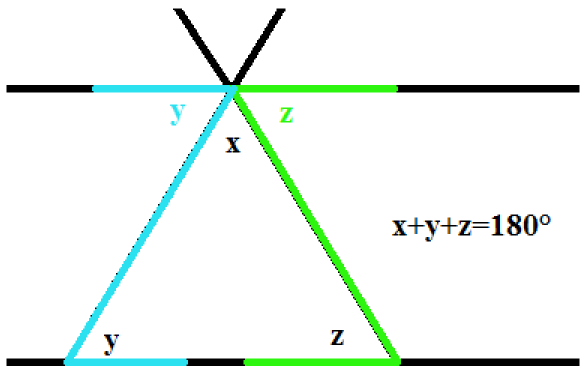


Using our knowledge of angles (interior, alternate, supplementary . . .) can we show why this is true? Why is is that with any given triangle the angles always add up to 180.

Lets try extending the lines and look at this again.

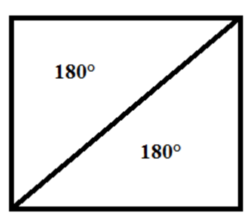


Now can you show the angles add up to 180?



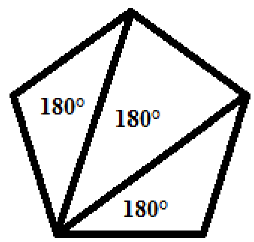
If we consider the angle $x + y$ we can say this angle is supplementary to z(we also could of said $x + z$ is supplementary to y. This means that $x + y + z$. Notice, we did this is unknown angles, this means that it works for any x, y and z.

What is the sum of angles in a parallelogram? Can you show this using triangles? The sum of the angles in a parallelogram is equal to 360 and we can show this using triangles. Draw a square, and from one corner draw a line to the opposite corner.



Each triangle has 180 degrees of interior angles, and there are two of them so $180+180 = 360$.

What about a pentagon? Start with a pentagon, chose a corner and draw a line to each corner that is not directly beside it.

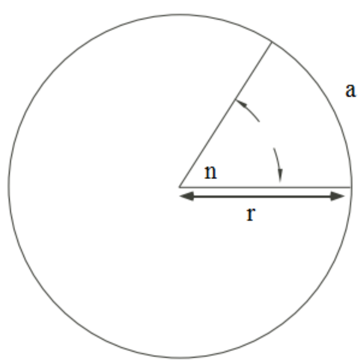


Using this same idea, we see that the sum of the angles inside a pentagon is
$180 + 180 + 180 = 540$. Can you think of how this might extend further? What would the sum of interior angles be in a heptagon(7 sides), what about a decagon(10 sides)?

Radians

Until now, all the angle we have seen were in degrees. However, angles like many other measurements can have different unit (similar to kilometers and miles). For angles, a commonly used unit is the radian. One radian is the angle formed when the arc length on a circle is equal to the radius. How many radians in one full circle?

Looking at the arc length formula
$a = \frac{n}{360} \times 2\pi r$ here n is angle made by the arc length.



In this formula we have the number 360. This is the number of degrees in a circle. To show how many radians we have in a circle lets replace that with a variable *rad*. Giving us
$a = \frac{n}{rad} \times 2\pi r$
Now n is the number of degrees, or in our case radians, and we defined 1 radian as then angle when the arc length is the same as the radius. In other words when $a = r$. For ease of

calculation lets make that be 1, $a = r = 1$. Our formula is now:

$1 = \frac{1}{rad} \times 2\pi \times 1$ Multiply both sides by rad
$rad = 2\pi$

Therefore there are 2π radians in a circle. This lead the equality $2\pi = 360°$

Conversion

We can use this equality to convert from degrees to radians and back. For example, convert 60°to radians. If we take the formula and divide by 360 on each side we get

$\frac{2\pi}{360} = 1$ Then we multiply each side by the desired angle
$\frac{2\pi \times 60}{360} = 60$ simplify
$\frac{2\pi}{6} = 60$
$\frac{\pi}{3} = 60$

So 60 degrees is equal to $\frac{\pi}{3}$ radians.

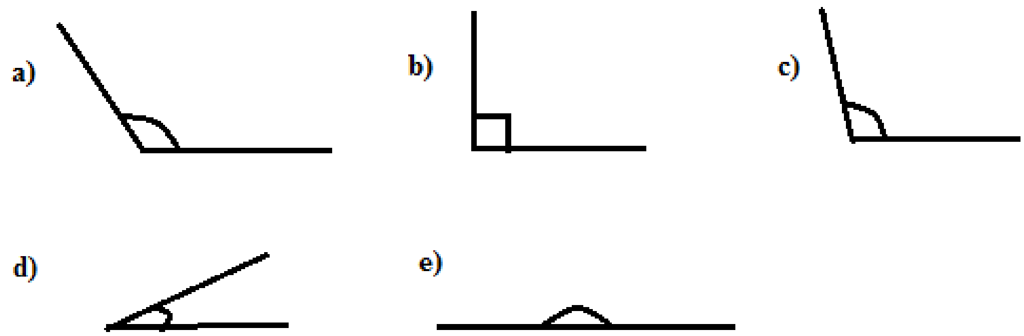
Going from radians to degrees we can do something similar. For example, convert $\frac{\pi}{4}$ to degrees. We start by dividing each side by 2 to get
$\pi = 180$ Now we want $\frac{\pi}{4}$ so we multiply each side by $\frac{1}{4}$
$\frac{\pi}{4} = \frac{180}{4}$ and simplify
$\frac{\pi}{4} = 45$

Examples Convert the following to either degrees or radians

1. π
2. $120°$
3. $\frac{4\pi}{3}$
4. $300°$
5. $\frac{12\pi}{6}$
6. $\frac{\pi}{18}$
7. $20°$

PROBLEMS

1. Identify the type of angles.

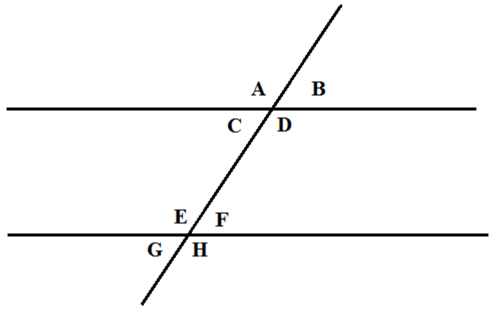


a) Obtuse   b) Right   c) Obtuse   d) Acute   e) Straight

2. What is a complementary angle? What is a supplementary angle? Two angles are complementary if they add up to 90°. Two angles are supplementary if they add up to 180°

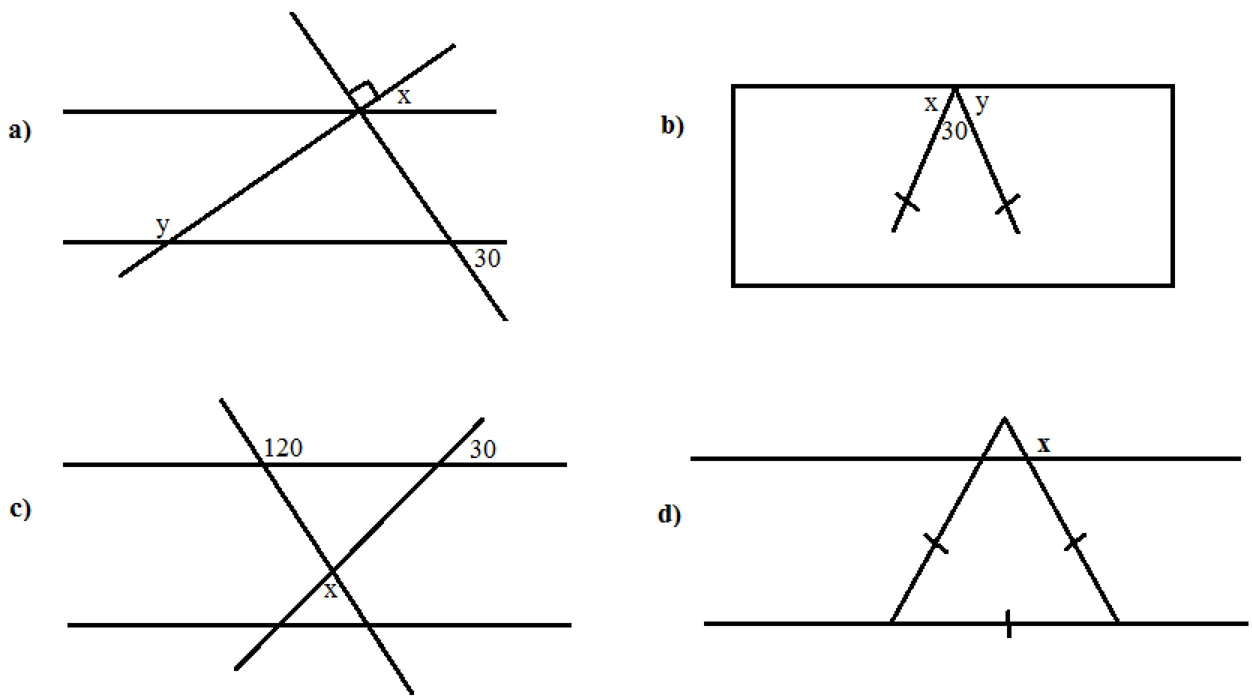
3. (a) What is the complementary angle of 30°? 60
   (b) What is the complementary angle of 17°? 73
   (c) What is the supplementary angle of 25°? 155
   (d) What is the supplementary angle of 111°? 69
   (e) What is the supplementary angle of 90°? 90
   (f) If angle w and x are complementary and y and z are complementary what are x and $(w + y + z)$? They are supplementary
   (g) If x is supplementary to 42°and y is complementary to 42°what is $x + y$? 186

4. List all pairs of angles that are supplementary to each other.



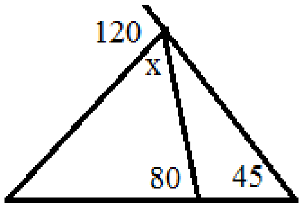
(A,B), (A,C), (A,G), (A,F),
(B,D), (B,H), (B,E),
(C,D), (C,E), (C,H) ,
(D,F), (D,G)
(E,F), (E,G),
(F,H),
(G,H)

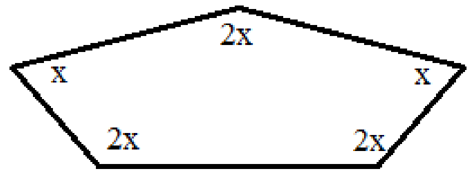
5. Find the missing angles.

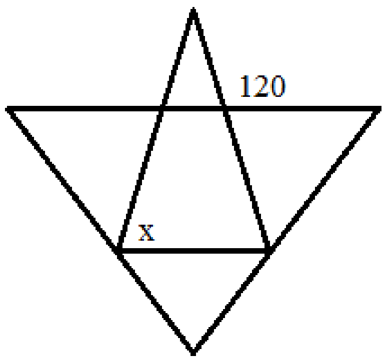


a) $x = 60$, $y = 120$ b) $x = 75$, $y = 75$ c) $x = 30$ d) $x = 120$

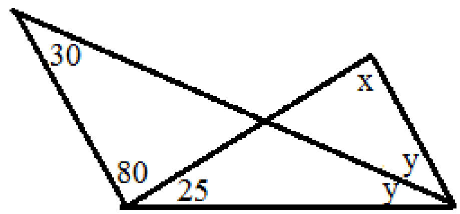
6. Find the missing angles.

a) 

b) 

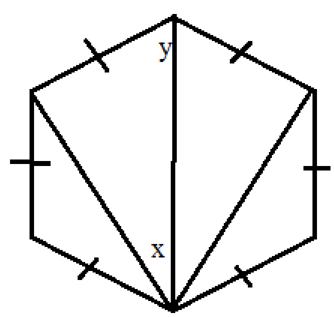
c) 

Note: Large upside down triangle is equilateral

d) 

a) $x = 25$ b) $x = 67.5$ c) $x = 60$ d) $x = 110, y = 22.5$

7. Find x and y in the following hexagon:



$x = 30, y = 60$

8. Two angles are supplementary to each other. One is 56 degrees larger then the other. What is the smallest angle.

62

9. You order a pizza and it is cut into 10 even pieces. If you were to measure the angle of a slice, what would it be in radians and in degrees?
   36°or $\frac{\pi}{5}$ rad

10. Convert

    (a) 3π 540°

    (b) 190° $\frac{19\pi}{18}$

    (c) $\frac{4\pi}{6}$ 120°

    (d) 270° $\frac{3\pi}{2}$

    (e) $\frac{8\pi}{9}$ 160°

    (f) $\frac{\pi}{18}$ 10°

    (g) 20° $\frac{\pi}{18}$

11. Find a pattern in the sum of angles inside a triangle, square, pentagon . . . Using this pattern What is the sum of interior angles in a 92 faced figure?
    The pattern is $(n-2) \times 180$ Therefore in a 92 faced figure, the sum of interior angles is:

$$\begin{aligned} &= (n-2) \times 180 \\ &= (92-2) \times 180 \\ &= (90) \times 180 \\ &= 16200 \end{aligned}$$

12. ** The ∠A is three times the size ∠B and ∠B is twice the size of ∠C. If ∠A and two times ∠C are supplementary what is ∠B?

    Here we have $A = 3 \times B$ and $B = 2 \times C$. Knowing A and C are supplementary we can write $A + 2C = 180$. Replace A and C and we get $3B + B = 180$. This implies that $B = 45$