UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

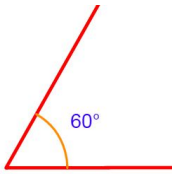
Grade 7/8 Math Circles

October 14/15, 2014

Angles

Angles

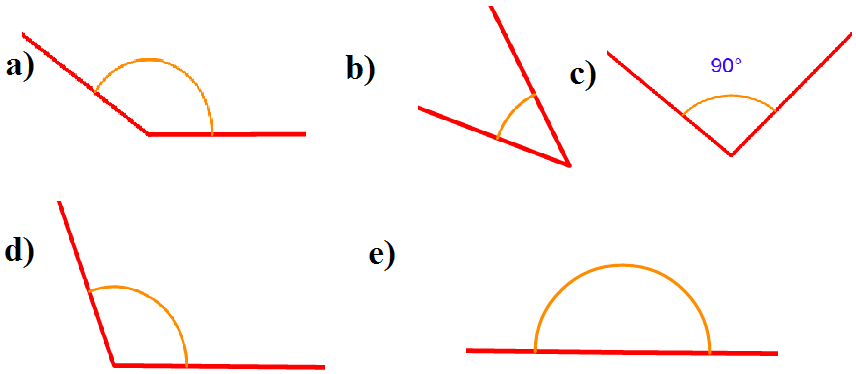
An **angle** is the space between two lines that intersect each other.



Terms and angles you should know:

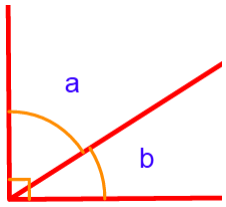
An **acute** angle is _____
A **right** angle is _____
An **obtuse** angle is _____
A **straight** angle is _____

Classify these angles as on of the above:



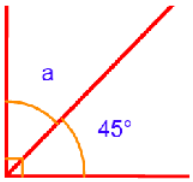
a) b) c) d) e)

Two angles are **complementary** if _____

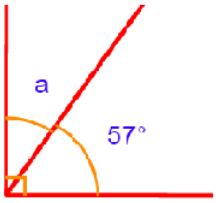


Find the complementary angle for the given angles(find a):

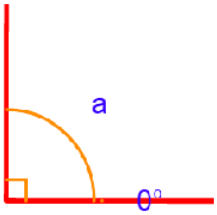
a)



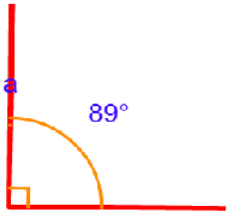
b)



c)

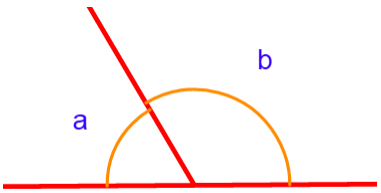


d)



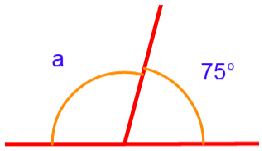
a) b) c) d)

Two angles are **supplementary** if _____



Find the supplementary angle for the given angles(find a):

a)



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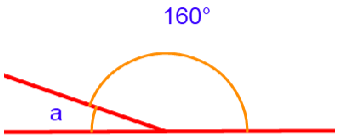
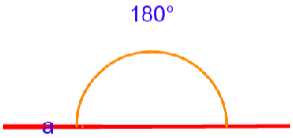
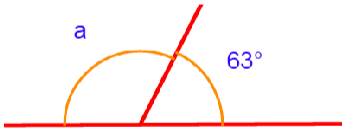


c)



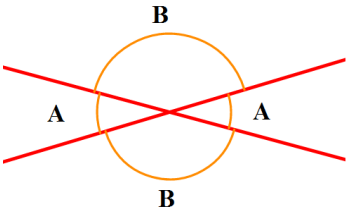
d)



a) b) c) d)

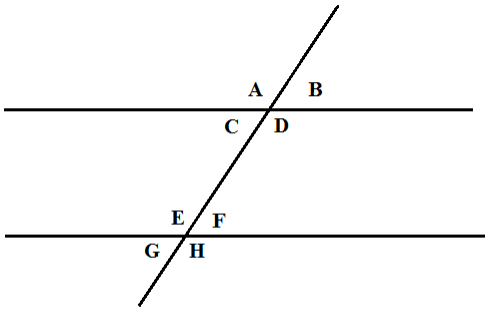
Trick to remember: _____

An **opposite** angle is _____

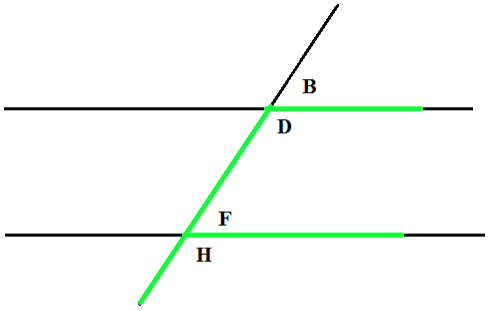


Can we explain this using our knowledge of supplementary angles?

A **transversal** is _____

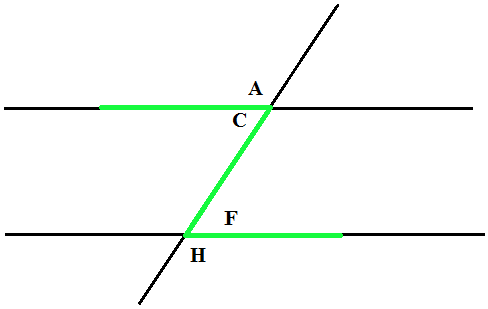


Corresponding angles are _____



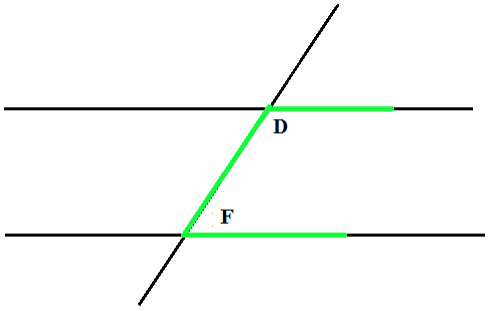
In the picture above there are 2 pairs of corresponding angles: D=H and B=F.

Alternate angles are _____



In the picture above there are 2 pairs of alternate angles: C=F and A=H.

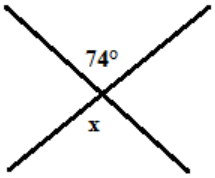
Interior angles are _____



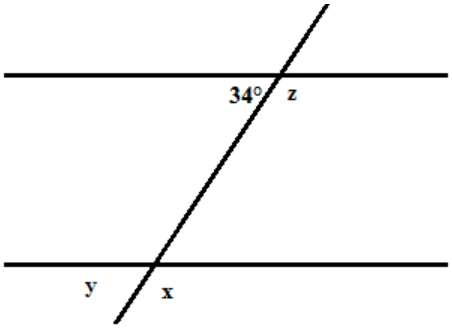
In the picture above D and F are interior angles. $D + F = 180$

Find the missing angles.

a)



b)



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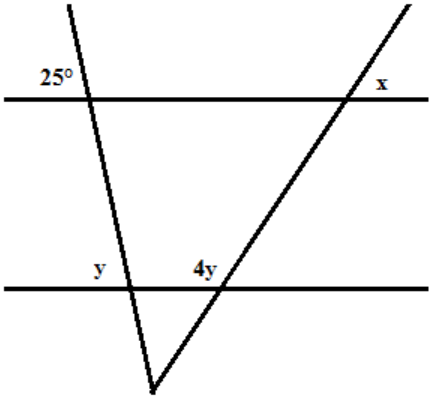

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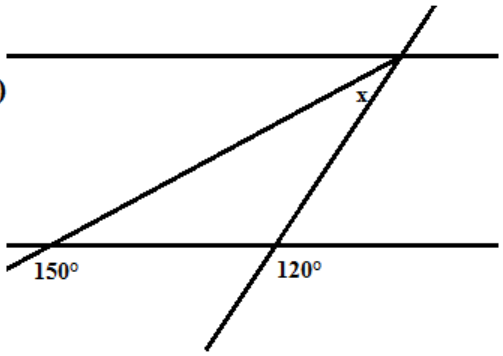
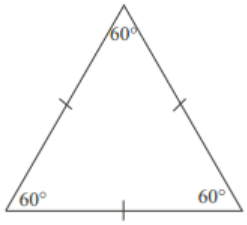


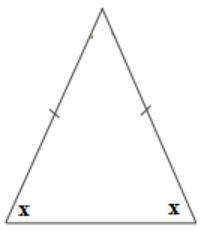
Triangles and More

First, lets talk about the 3 types of triangles.

Equilateral: _____

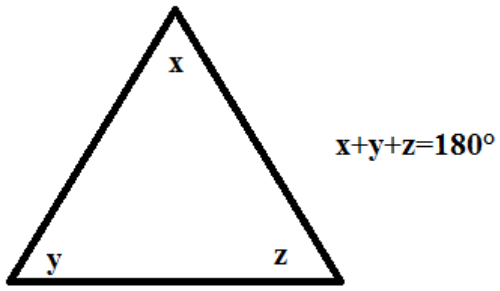


Isosceles: _____



Scalene: _____

What is the sum of the interior angles of a triangle? 180. Any triangle, regardless of size has 3 interior angles that add up to 180.



Using our knowledge of angles (interior, alternate, supplementary . . .) can we show why this is true? Why is is that with any given triangle the angles always add up to 180.

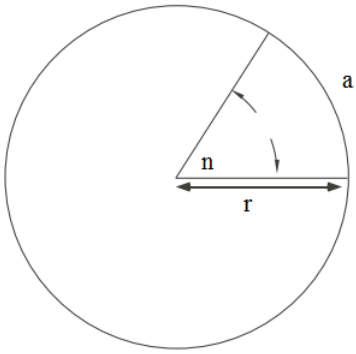
What is the sum of angles in a parallelogram? Can you show this using triangles? The sum of the angles in a parallelogram is equal to 360 and we can show this using triangles. Draw a square, and from one corner draw a line to the opposite corner.

What about a pentagon? Start with a pentagon, chose a corner and draw a line to each corner that is not directly beside it.

Radians

Until now, all the angle we have seen were in degrees. However, angles like many other measurements can have different unit (similar to kilometers and miles). For angles, a commonly used unit is the radian. One radian is the angle formed when the arc length on a circle is equal to the radius. How many radians in one full circle?

Looking at the arc length formula
$a = \frac{n}{360} \times 2\pi r$ here n is angle made by the arc length.



In this formula we have the number 360. This is the number of degrees in a circle. To show how many radians we have in a circle lets replace that with a variable rad. Giving us $a = \frac{n}{rad} \times 2\pi r$

Now n is the number of degrees, or in our case radians, and we defined 1 radian as then angle when the arc length is the same as the radius. In other words when $a = r$. For ease of calculation lets make that be 1, $a = r = 1$. Our formula is now:

$1 = \frac{1}{rad} \times 2\pi \times 1$ Multiply both sides by rad
$rad = 2\pi$

Therefore there are 2π radians in a circle. This lead the equality 2π radians $= 360°$

Conversion

We can use this equality to convert from degrees to radians and back. For example, convert 60°to radians. If we take the formula and divide by 360 on each side we get

$\frac{2\pi}{360} = 1$ Then we multiply each side by the desired angle
$\frac{2\pi \times 60}{360} = 60$ simplify
$\frac{2\pi}{6} = 60$
$\frac{\pi}{3} = 60$

So 60 degrees is equal to $\frac{\pi}{3}$ radians.

Going from radians to degrees we can do something similar. For example, convert $\frac{\pi}{4}$ to degrees. We start by dividing each side by 2 to get
$\pi = 180$ Now we want $\frac{\pi}{4}$ so we multiply each side by $\frac{1}{4}$

$\frac{\pi}{4} = \frac{180}{4}$ and simplify

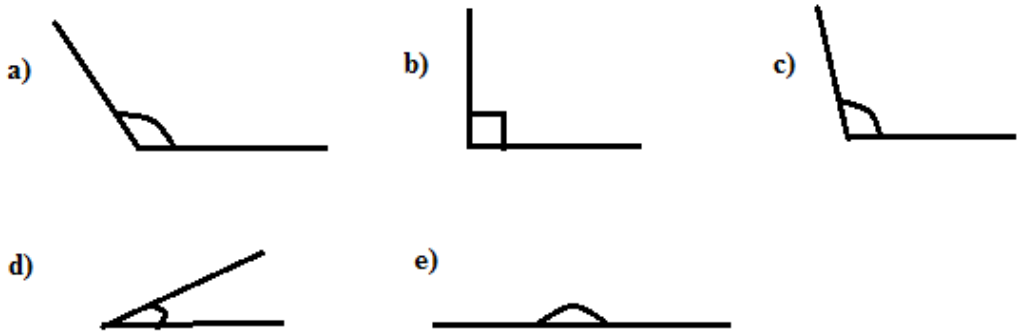
$\frac{\pi}{4} = 45$

Examples Convert the following to either degrees or radians

1. π
2. 120°
3. $\dfrac{4\pi}{3}$
4. 300°
5. $\dfrac{12\pi}{6}$
6. $\dfrac{\pi}{18}$
7. 20°

PROBLEMS

1. Identify the type of angles.

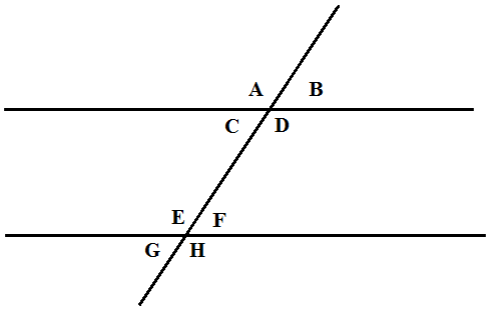


a) b) c) d) e)

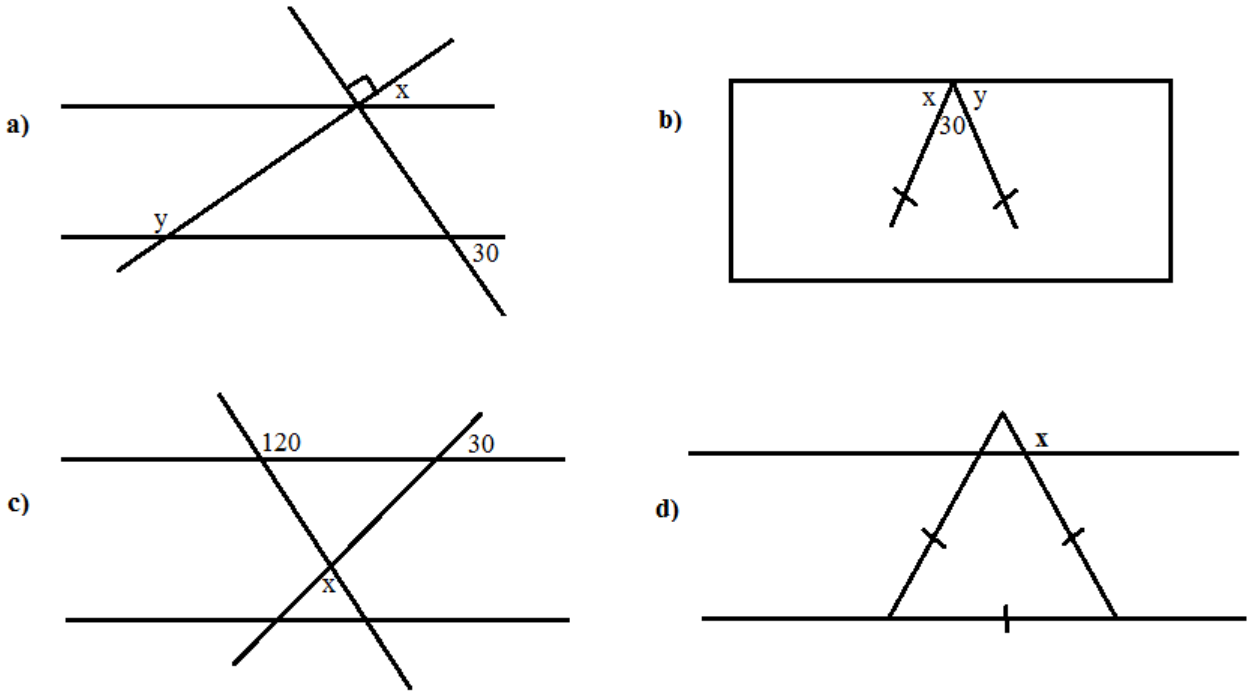
2. What is a complementary angle? What is a supplementary angle?

3. (a) What is the complementary angle of 30°?
   (b) What is the complementary angle of 17°?
   (c) What is the supplementary angle of 25°?
   (d) What is the supplementary angle of 111°?
   (e) What is the supplementary angle of 90°?
   (f) If angle w and x are complementary and y and z are complementary what are x and $(w + y + z)$?
   (g) If x is supplementary to 42°and y is complementary to 42°what is $x + y$?

4. List all pairs of angles that are supplementary to each other.

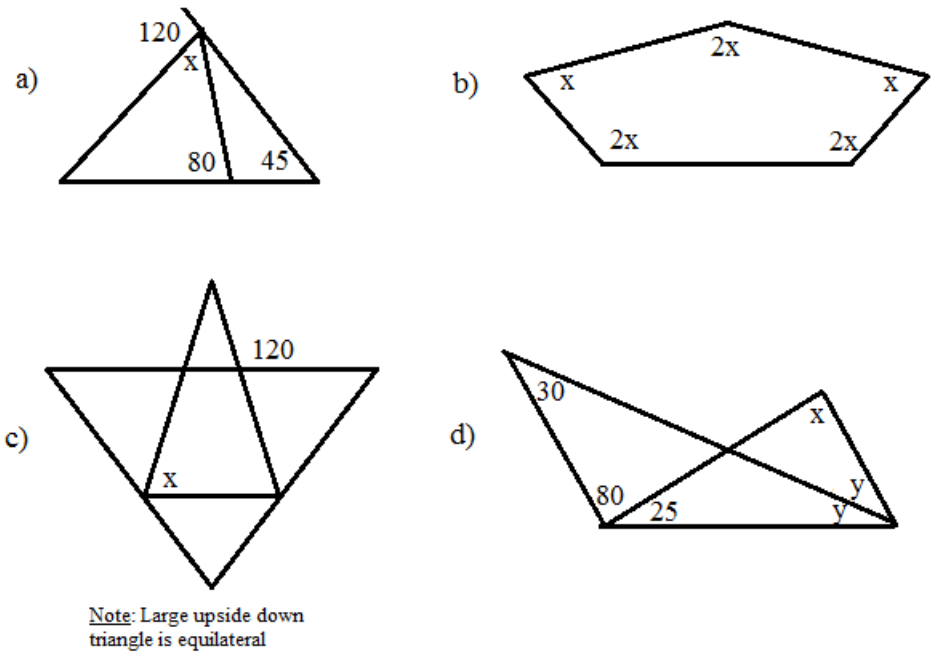


5. Find the missing angles.



a) b) c) d)

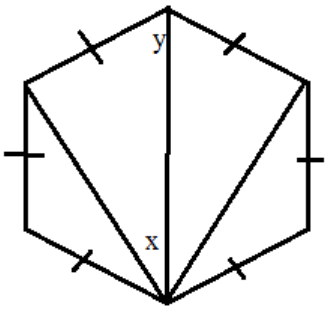
6. Find the missing angles.



Note: Large upside down triangle is equilateral

a) b) c) d)

7. Find x and y in the following hexagon:



8. Two angles are supplementary to each other. One is 56 degrees larger then the other. What is the smallest angle.

9. You order a pizza and it is cut into 10 even pieces. If you were to measure the angle of a slice, what would it be in radians and in degrees?

10. Convert

    (a) 3π

    (b) 190°

    (c) $\frac{4\pi}{6}$

    (d) 270°

    (e) $\frac{8\pi}{9}$

    (f) $\frac{\pi}{18}$

    (g) 20°

11. Find a pattern in the sum of angles inside a triangle, square, pentagon ... Using this pattern What is the sum of interior angles in a 92 faced figure?

12. he $\angle$A is three times the size $\angle$B and $\angle$B is twice the size of $\angle$C. If $\angle$A and two times $\angle$C are supplementary what is $\angle$B?