Math Circles
29 October 2014
Recurrences, Part 1
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EXERCISES

1. Prove by induction that the sum of the first n perfect squares is $\frac{n(n+1)(2n+1)}{6}$. What form of induction did you use?

2. Find and prove a closed form for $a_n = 2\sqrt{a_{n-1}}$ with $a_0 = 2$.

3. (a) Solve the recurrence $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$.
   (b) Redo part (a), except this time, change $a_0 = 10$ and $a_1 = 4$.
   (c) Redo part (a), except this time, change $a_0 = a$ and $a_1 = b$.

4. Solve the recurrence $a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$ with $a_0 = 8$, $a_1 = 6$ and $a_2 = 26$.

5. Solve the recurrence $a_n = 6a_{n-1} - 9a_{n-2}$ with $a_0 = 1$ and $a_1 = 6$.

6. Solve the recurrence $s_k = as_{k-1}$ for any value of a. Using that solution, solve $s_k = as_{k-1} + 1$.

7. Solve $a_n = 2a_{n-1} - a_{n-2} + 2^n$ with $a_0 = 1$ and $a_1 = 2$.

8. If $\varphi = \frac{1+\sqrt{5}}{2}$, find $-\frac{1}{\varphi}$. Then, find $1 - \varphi$. Be amazed.