Intermediate Math Circles
April 1 2015
Analytic Geometry II

2. Quick Exercise

For the points $A(-3, 5)$ and $B(7, 2)$, determine the distance AB, the midpoint of AB, the slope of AB and the slope of a line perpendicular to AB.

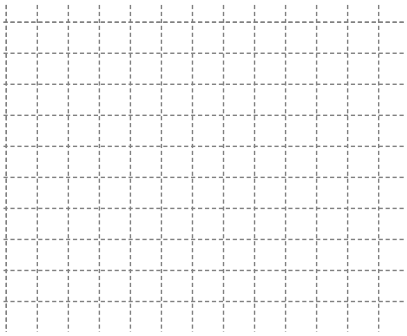
3. Proofs in Analytic Geometry

The final answer here is NOT the important thing - presentation of a full and complete solution is.

The diagram (sketch) supports the solution BUT is not the solution.

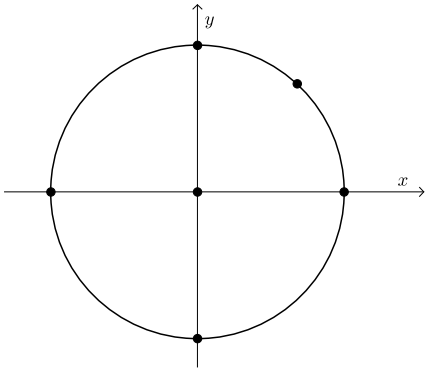
The setup of an analytic proof is extremely important.

(i) Prove, using analytic geometry, that the diagonals of a square right bisect each other.



(ii) Prove that the angle inscribed in a semi-circle is 90°.

Proof 1:



Proof 2:

4. Equations of Lines

To determine the equation of a line you need to know the slope of the line and a point on the line OR you need to know two points on the line.

In grade 9 you generally learn two forms of the equation of a line:

- Slope - Intercept Form

- "Standard" Form

There are other forms of the equation of a line that you may not be familiar with. We will consider one more here.

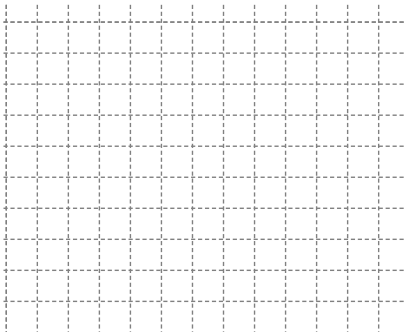
- Point-Slope Form

(iii) If $(4, -2)$ is a point on a line with slope 7, then determine the equation of the line.

If (x_0, y_0) is a point on a line with slope m, then the equation of the line can be written

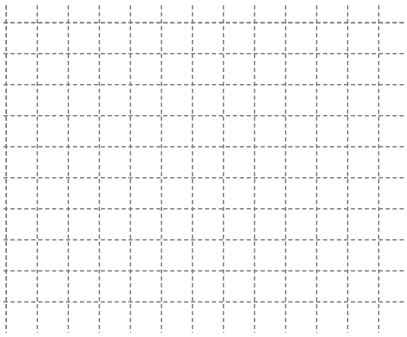
4. Equations of Lines (continued)

(iv) Determine the equation of the line through the point $A(-1, 5)$ that is perpendicular to the line $y = \frac{-3}{2}x + 5$. Express your answer in standard form.



6. Problems Involving Equations of Lines

(v) Two telephone poles are 12 m apart. One pole is 8 m tall and the other is 10 m tall. A wire is strung from the top of each pole to the bottom of the other pole. The wires cross somewhere between the two poles. How high above the ground do the wires meet?



7. Distance From A Point To A Line

(vi) Determine the shortest distance from the point $P(1, 7)$ to the line $2x + y - 4 = 0$.

Since finding the distance from a point to a line is basically an algebraic problem, we can develop a general formula for the distance from a point to a line.

The distance from the point $P(x_1, y_1)$ to the line $Ax + By + C = 0$ is

We will not prove this here because of time. However, if you do a Google search on "distance of a point to a line proof" you will find an excellent proof that you could work through. Otherwise, in the Calculus Vectors course, a very efficient proof is presented.

We will verify our answer to the previous problem using the formula.
Find the distance from the point $P(1, 7)$ to the line $2x + y - 4 = 0$.