Intermediate Math Circles
Analytic Geometry II
Problems April 1 2015

1. Prove, using analytic methods, that the diagonals of a parallelogram bisect each other.
2. Prove, using analytic methods, that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and one-half the length of the third side.
3. Determine the distance from the point $Q(-3, 5)$ to the line $2x - 7y + 1 = 0$.
4. Calculate the coordinates of the foot of the perpendicular from the point $(2, -6)$ to the line $x - 3y - 2 = 0$.
5. In $\triangle ABC$, with vertices $A(2, 1)$, $B(12, 6)$ and $C(0, 10)$, an altitude is drawn from C touching AB at D. Determine the length of the altitude CD. Develop two different solutions.
6. A point P is chosen on the line $y = 2x + 3$ and a point Q is chosen on $y = -x + 2$. If the midpoint M of the line segment PQ is $(2, 5)$, determine the coordinates of P and Q.

Answers

3. $\dfrac{40}{\sqrt{53}} = \dfrac{40\sqrt{53}}{53} \doteq 5.5$
4. $\left(\dfrac{1}{5}, \dfrac{-3}{5}\right)$
5. $4\sqrt{5}$
6. $P(3, 9)$ and $Q(1, 1)$