Intermediate Math Circles
February 11, 2015
Contest Preparation II

WARM-UP: Not Your Ordinary Magic Square!

A 4 by 4 “anti-magic” square is an arrangement of the numbers 1 to 16 inclusive in a square so that the totals of each of the four rows and four columns and two main diagonals are ten consecutive numbers in some order. The diagram shows an incomplete “anti-magic” square. When the chart is completed what number will replace α?

4	5	7	14
6	13	3	α
11	12	9	
10			

Upcoming Contests

Tuesday, February 24, 2015	Pascal	Cayley	Fermat
Thursday, April 16, 2015	Fryer	Galois	Hypatia
November 2015	Canadian Intermediate Math Contest		
November 2015	Beaver Computing Challenge		

Tonight's problems have been taken mainly from past CEMC Contests.

Problem #1

Four years ago, Daryl was three times as old as Joe was. In five years, Daryl will be twice as old as Joe will be. How old is Daryl now?

Problem #2

Determine the number of positive divisors of 18 800 that are divisible by 235.

Problem #3

What is the area of $\triangle ABC$ with vertices $A(3,-1)$, $B(5,1)$ and $C(8,7)$?

Problem #4

The average of three consecutive multiples of 3 is a. The average of four consecutive multiples of 4 is $(a+27)$. The average of the smallest and largest of these seven integers is 2022. Determine the value of a.

Problem #5

At last week's hockey game involving the Waterloo Blueberries, the price of a platinum ticket was \$25, the price of a gold ticket was \$10, the price of a silver ticket was \$5, and the price of a bronze ticket was \$1.

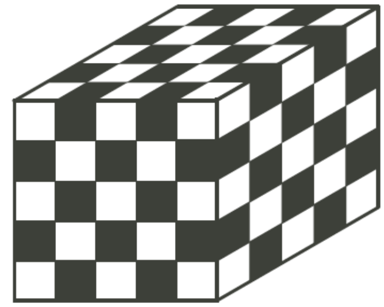
a) Jim buys 5 platinum tickets, 2 gold tickets and 3 silver tickets. Determine the average cost of the tickets that Jim buys. Explain how you obtained your answer.

b) Mike buys 8 tickets whose average cost is \$12. He then buys 5 more platinum tickets. What is the new average cost of the tickets that he has bought? Explain how you obtained your answer.

c) Ophelia buys 10 tickets with an average cost of \$14. Suppose that she buys n more platinum tickets. The new average cost of the tickets that she has bought is \$20. What is the value of n? Explain how you obtained your answer.

Problem #6

a) Each face of a 5 by 5 by 5 wooden cube is divided into 1 by 1 squares. Each square is painted black or white, as shown. Next, the cube is cut into 1 by 1 by 1 cubes. How many of these cubes have at least two painted faces?



b) A $(2k+1)$ by $(2k+1)$ by $(2k+1)$ cube, where k is a positive integer, is painted in the same manner as the 5 by 5 by 5 cube with white squares in the corners. Again, the cube is cut into 1 by 1 by 1 cubes.

i) In terms of k, how many of these cubes have exactly two white faces?

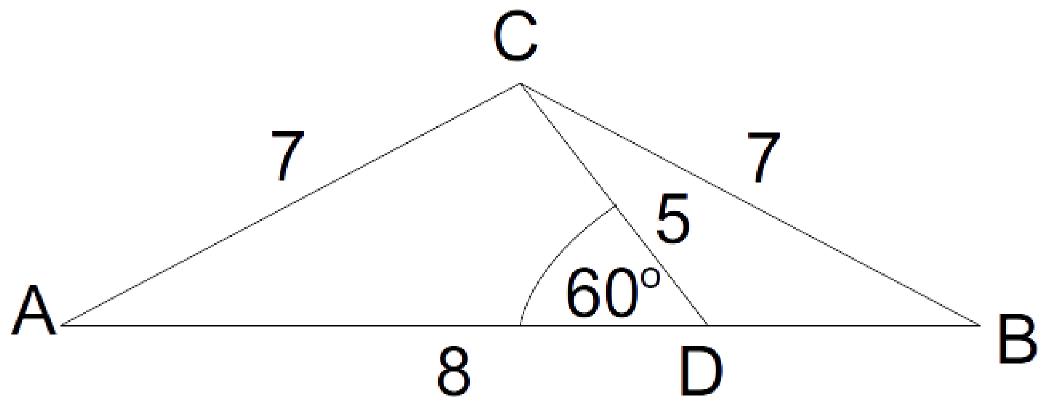
ii) Prove that there is no value of k for which the number of cubes having at least two white faces is 2006.

Problem #7

The number 8 is the sum and product of the numbers in the collection of four positive integers $\{1, 1, 2, 4\}$, since $1 + 1 + 2 + 4 = 8$ and $1 \times 1 \times 2 \times 4 = 8$. The number 2007 can be made up from a collection of n positive integers that multiply to 2007 and add to 2007. What is the smallest value of n with $n > 1$?

Problem #8

In the diagram $\triangle ABC$ is isosceles with $AC = BC = 7$. Point D is on AB with $\angle CDA = 60°$, $AD = 8$ and $CD = 5$. Determine the length of BD.



Problem #9

A robot is programmed to find a target (the green field marked with X) on a map of square fields. The robot has its movements programmed as follows:

- The robot moves straight forward until it reaches an obstacle (black field) or the edge of the map.
- When reaching an obstacle or the edge of the map, the robot turns right by 90°.
- When the robot moves out of a field, the field becomes a black obstacle.

The arrows on the maps below show the starting position as well as the starting direction of the robot.
On which map does the robot NOT eventually reach the target (green field marked with X)?