Intermediate Math Circles
February 18, 2015
Solutions

Problem Set 1

1. Fill in the blanks in the following patterns. (there may be more than one answer!)

   (a) $0, 3, \underline{8}, 15, \underline{24}, \underline{35}, 48$

   (b) $\uparrow, \nearrow, \rightarrow, \searrow, \underline{\downarrow}, \underline{\swarrow}$

   (c) $2, 5, 7, 12, 19, 31, \underline{50}, \underline{81}, \underline{131}$

   (d) $\frac{1}{2}, \frac{2}{4}, \frac{3}{8}, \frac{4}{16}, \underline{\frac{5}{32}}, \underline{\frac{6}{64}}, \underline{\frac{7}{128}}$

2. Identify the following sequences as Arithmetic, Geometric or Neither. Find t_{10} and t_n for each.

   (a) $3, 7, 11, 15, 19, \cdots$
   Arithmetic . $t_{10} = 39$, $t_n = 4n - 1$

   (b) $3, 33, 333, 3333, \cdots$
   Neither. $t_{10} = 3333333333$, $t_n = n$ 3's in a row

   (c) $2, 6, 18, 54, \cdots$
   Geometric, $t_{10} = 2 \times 3^9$, $t_n = 2 \times 3^{n-1}$

3. Find 5 well defined sequences that have first term 2 and have a later term 10.
   $2, 10, 18, 26, \cdots$
   $2, 10, 50, 250. \cdots$
   $2, 4, 6, 8, 10, \cdots$
   $2, 6, 10, 14, \cdots$
   $2, 3, 4, 5, 6, 7, 8, 9, 10, \cdots$
   And there are many more answers.

Problem Set 2

1. Starting at 9 and counting by 7's, a student counts 9, 16, 23, etc. What is the 15^{th} number the students says?

   $t_n = 7n + 2$ so $t_{15} = 107$

2. Find the general formula for the sequence defined by the recursive formula $t_1 = -3$ and $t_n = t_{n-1} + 5$ for $n > 1$.

   The sequence is $-3, 2, 7, 12, \cdots$ so the general term is $t_n = 5n - 8$.

3. Find the recursive formula for the sequence defined by the general formula $t_n = 2^n + 5$.

   The sequence is $7, 9, 15, 23, \cdots$ so the recursive formula is $t_1 = 7$, $t_n = t_{n-1} + 2^{n-1}$ for $n > 1$.

4. There are 23 bacteria cells in a laboratory dish at 1 pm. The bacteria triple in number every 15 minutes.

   (a) How many bacteria are there at 2:30 pm?

   The sequence is $23, 69, 207, 621, 1863, 5589, 16767, \cdots$ so there are 16767 bacteria at 2:30 pm.

   (b) The clock in the lab chimes once at 1:00, twice at 2:00, three times at 3:00 and so on. What is the first number of chimes you will hear when there are at least 1 000 000 bacteria?

   The general term for this sequence is $t_n = 23 \times 3^{n-1}$. If I put 10 in for n I am under 1000000 while 11 puts me over.
   Remember that t_1 is at 1:00 and t_2 is at 1:15, then t_{10} is at 3:30 and t_{11} is at 3:45. That means you will have 1000000 bacteria sometime between 3:30 and 3:45 but the first chimes will be at 4:00 and there will be 4 chimes.

5. The numbers 2,5,8,11,14,$\cdots$ are written in order in a book, one hundred numbers to a page beginning on page one. The number 11 111 will be found on what page?

   The general term for the sequence is $t_n = 3n - 1$. Solve $11111 = 3n - 1$ to get $n = 3704$. Then 11111 is the 3704th term and will be on the 37th page.

6. The numbers in the sequence 2, 7, 12, 17, 22, . . . increase by fives.
   The numbers in the sequence 3, 10, 17, 24, 31, . . . increase by sevens.
   The number 17 appears in both sequences. What is the next number which appears in both sequences?

   52 is the answer but you should note this continues every 35 as 35 is the LCM of 7 and 5.

7. In a sequence of six numbers, the first number is 4 and the last number is 47. Each of the numbers after the second is equal to the sum of the previous two numbers. Determine the sum of the six numbers.

   The sequence is $4, 7, 11, 18, 29, 47$ so the sum of the 6 numbers is 116.

8. The sum of the first n terms of a sequence is $n(n + 1)(n + 2)$. Determine the 10th term of the sequence.

   The sum of the first 10 terms is $10(11)(12) = 1320$.
   The sum of the first 9 terms is $9(10)(11) = 990$.
   Then the 10th term must be the difference $1320 - 990 = 330$.