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Grade 6 Math Circles

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Sets

Some of These Things Are Not Like the Others

When given a lot of information, it is important that we organize it so that we can better understand this information. For example, when grocery shopping, you would want to organize the list so that all of the fruits are together, and you can pick them up at the same time.

Given our grocery list, we can make a list of the fruits so that it is easier to shop.

Strawberries	Cereal	Raspberries	Cherries
Chocolate	Bagels	Eggs	Milk
Carrots	Bananas	Broccoli	Butter
Red Apples	Bread	Red Peppers	

Fruits:

Strawberries, Apples, Bananas, Cherries, Raspberries

Now make a list of all the red foods (1) and then another with all the foods that start with the letter "B" (2).

1.Strawberries, Apples, Raspberries, Red Peppers, Cherries

2.Bagels, Bananas, Bread, Broccoli, Butter

Notice that we can write a list that follows any rule!

What is a Set?

A *set* is a grouping of information that follows a specific rule. The members of the set are called its *elements*. We use the curly brackets {} when we are writing a set.

So, when making a list of fruits from our grocery list, we can write:
{Strawberries, Apples, Bananas, Raspberries, Cherries}

We can also write sets of numbers. For example, the set of even numbers between 1 and 11 is: {2,4,6,8,10}
The order of the elements in a set does not matter. Also, there can be no duplicates in a set (there are no repeating elements).

Naming Sets

Sometimes saying "the set of all even numbers" is too long. (Plus, mathematicians like to be lazy!).To shorten this, we can put the rule inside of the brackets. So, instead of saying "the set of all even numbers", we have {even numbers}.
We can also give a set a name. This can be anything you want! . However, we usually name sets with a capital letter. If we want {even numbers} to be called "H", we say
H = {even numbers}.

Set Notation

To say that something is in a set, we use the Greek letter epsilon, $\in$.
We can translate $\in$ into English as "is an element of". Going back to our example with the groceries, we can say that apples $\in$ {fruits},
which means "apples are an element of the set of fruits."
However, if an element is not in a set, we cross out the epsilon.
For example, carrots $\notin$ {fruit}.

Which symbol should we see in the following situations? :
1. Soccer $\in$ {Team sports}

2. Running $\notin$ {Team sports}

Size of Sets

The number of elements in a set is known as its *size* or its *cardinality*.
To find the size of a set, just count the number of elements inside. So, if I have a set {Favourite movies} = {Harry Potter, Hunger Games, Star Wars, Lord of the Rings}, then the size of this set is 4.
We write |{Favourite movies}| = 4

However, sets can also be empty. To show the empty set, we use the symbol $\emptyset$
If I didn't like movies, I could say {Favourite movies} = $\emptyset$, and |{Favourite movies}| = 0

What is the size of {even numbers}?
Well, if we count all of the even numbers, we will be here all night! There is an *infinite* amount of even numbers.
So, we say |{even numbers}| = ∞, which means "infinity", or never-ending.
Find the size of the following sets:
1. |{Fruits}| (from the grocery list)

5

2. |{Months with 32 days}|

0

Equality of Sets

Two sets are *equal* if they have the same elements.

Remember: order does not matter, and you can't repeat any items in a set!

Are the following sets equal?

1. {soccer, hockey, basketball} and {hockey, basketball, soccer, tennis}

These sets are not equal

2. {all of the food in the grocery store ordered by colour} and
{all of the food in the grocery store ordered by weight}

These sets are equal

Unions of Sets

Consider 2 sets:

A = {team sports} = {hockey, soccer, football, basketball} and

B = {individual sports} = {running, swimming, biking}.

The *union* of sets A and B is the set of elements in A *or* B or both.

We write union with the symbol $\cup$.

So, A $\cup$ B = {hockey, soccer, football, basketball, running, swimming, biking}.

The easiest way to remember union is to remember the word **or**.

Let's go back to our grocery list:

Strawberries	Cereal	Raspberries	Cherries
Chocolate	Bagels	Eggs	Milk
Carrots	Bananas	Broccoli	Butter
Red Apples	Bread	Red Peppers	

1. Find {red foods} $\cup$ {fruits}.

{Strawberries, apples, raspberries, red peppers, cherries, bananas}

2. Find {red foods} $\cup$ {fruits} $\cup$ {foods that start with the letter "B"}.

{strawberries, apples, raspberries, red peppers, cherries, bananas, bagels, bread, butter, broccoli}

Is the union of these three sets any different from the union of the two sets in #1?

No, we still take all elements in set A or set B or set C, or two of A, B and C, or all 3.

Intersection of Sets

Consider the sets:

A = {apples, bananas, oranges, strawberries} and

B = {bananas, grapes, strawberries, nectarines}.

The *intersection* of sets A and B is the set of elements in A *and* B.

We write intersection with the symbol $\cap$.

So, $A \cap B$ = {bananas, strawberries}.

The easiest way to remember intersection is to remember the word **and**.

For the following examples, consider the sets B = {Boys} = {Sachin, Ryan, Josh, Ben},

G = {Girls} = {Sarah, Hannah, Hayley, Jennifer}, and

R = {Runners} = {Ryan, Sachin, Hannah, Hayley}.

1. What is the set {Girls} $\cap$ {Boys} ?

$\emptyset$

2. What is the set {Boys} $\cap$ {Runners} ?

{Ryan, Sachin}

Subsets

A *subset* is a set contained in another set. We write subset as $\subset$.

For example, both {Boys} and {Girls} are subsets of {Class}. So, we write {Boys} $\subset$ {Class} and {Girls} $\subset$ {Class}. If a set is not a subset of another set, we write $\not\subset$.

So, we write {Class} $\not\subset$ {Boys}.

An easy way to remember the subset is sign is to think of the letter "C": $\subset$ looks like "C" for "contained". We can say "Boys are contained in class".

Universal Sets

The *universal set* is the largest possible set.

We write the universal set with the symbol $\mathbb{U}$.

From our example, $\mathbb{U}$ = C = {Sachin, Ryan, Josh, Ben, Hannah, Hayley, Jennifer, Sarah}.

1. What is the universal set of months?

{All 12 months}

2. What is the universal set of positive whole numbers less than 6?

{1,2,3,4,5}

Complements of Sets

Consider a class of students:

C = {class} = {Sachin, Ryan, Josh, Ben, Sarah, Hannah, Hayley, Jennifer}.

Recall the students who are runners:

R = {Runners} = {Ryan, Sachin, Hannah, Hayley}

The *complement* of a set R is the set of all the students who are not in R.

We write the complement of R as $\overline{R}$.

In our example, $\overline{R}$ = {Sarah, Jennifer, Josh, Ben}.

The easiest way to remember complement is to remember the word **not**.

1. Given {Months} and T = {Months ending in "y"}, what is $\overline{T}$?

{March, April, June, August, September, October, November, December}

2. Given our class of students from above, what is $\overline{R} \cap$ R?

$\emptyset$

Now let's try combining unions, intersections and complements:

Here are our sets again:

$\mathbb{U}$ = {Class} = {Sachin, Ryan, Josh, Ben, Sarah, Hannah, Hayley, Jennifer}.

B = {Boys} = {Sachin, Ryan, Josh, Ben},

G = {Girls} = {Sarah, Hannah, Hayley, Jennifer},

R = {Runners} = {Ryan, Sachin, Hannah, Hayley}.

1. Find G $\cap$ B.

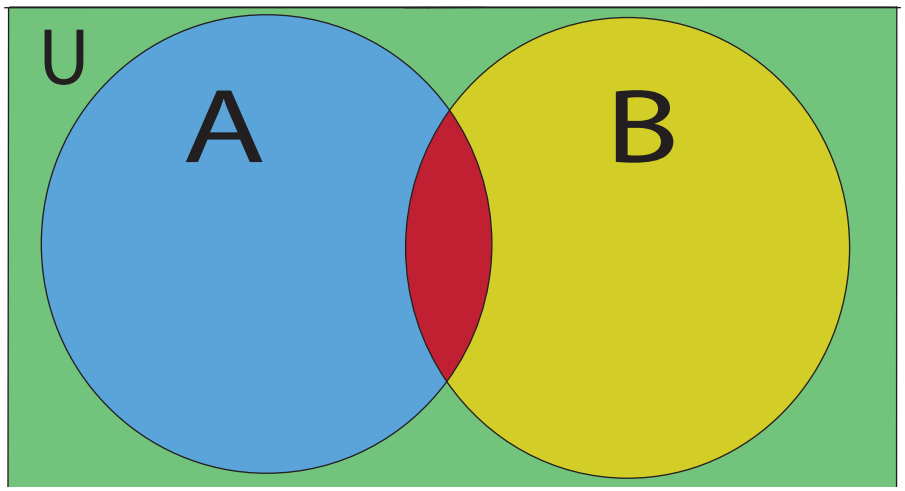
$\emptyset$

2. Find B $\cap$ $\overline{R}$.

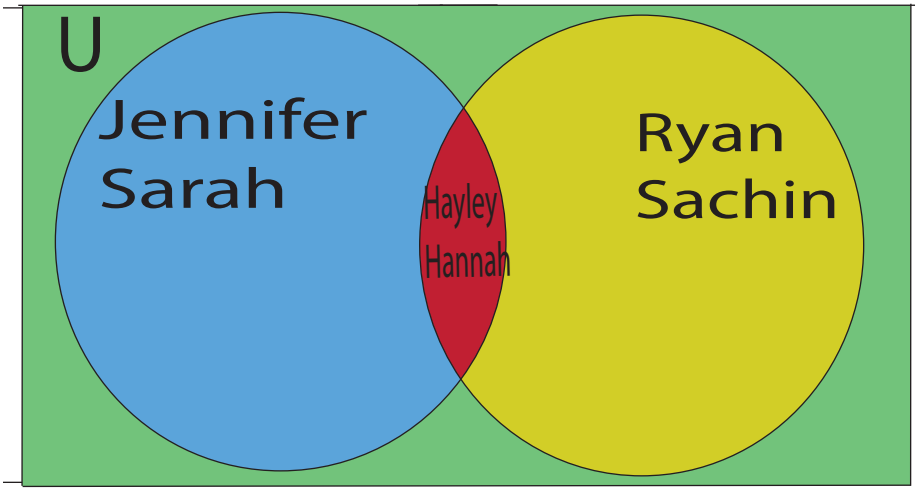
{Josh, Ben}

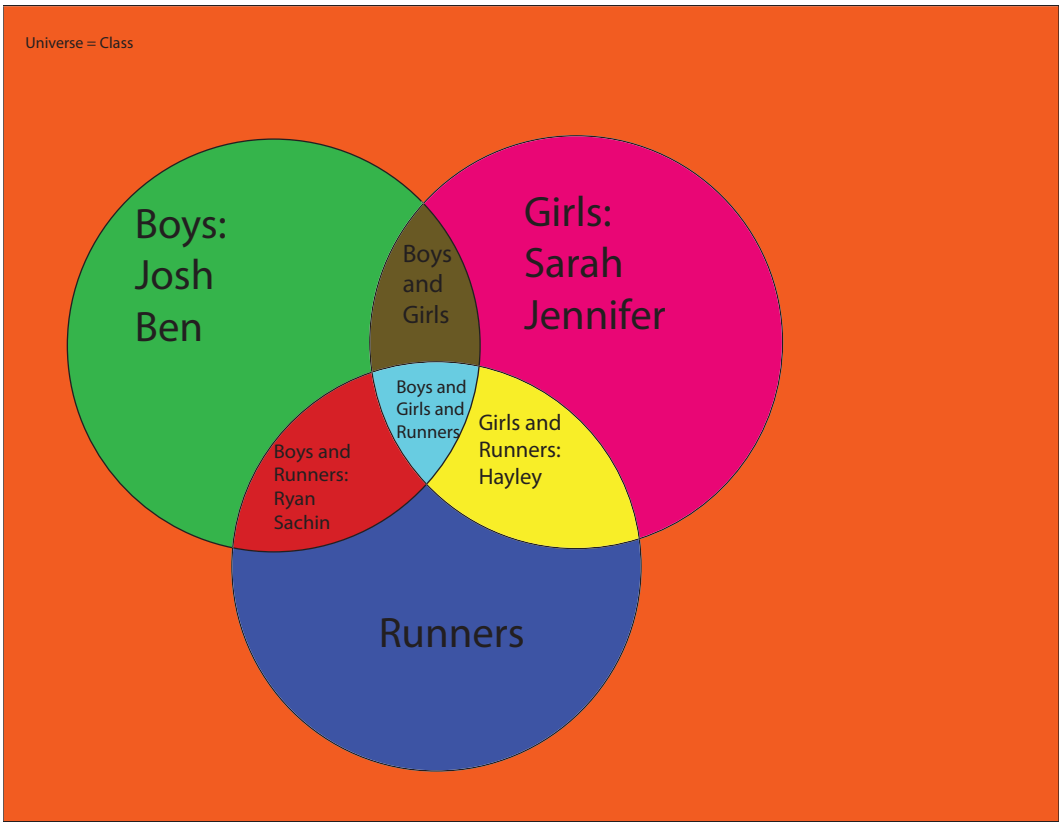
Using Venn Diagrams

An easy way to visualize union, intersection, complement and universe is by drawing Venn diagrams.



Both A and B represent sets.
The region where the circles overlap represents the intersection, $A \cap B$.(A and B)
The region covered by one or both circles represents the union, $A \cup B$.(A or B)
The entire rectangle represents the universal set, $\mathbb{U}$.
The region outside of A and B and inside the rectangle represents the complement of A and B (not A nor B)
Using the sets:
$\mathbb{U}$ = {Class} = {Sachin, Ryan, Josh, Ben, Sarah, Hannah, Hayley, Jennifer}.
B = {Boys} = {Sachin, Ryan, Josh, Ben},
G = {Girls} = {Sarah, Hannah, Hayley, Jennifer},
R = {Runners} = {Ryan, Sachin, Hannah, Hayley}.
Draw a Venn diagram for the following:
1. $G \cap R$
2. $B \cap G \cap R$


U
Jennifer
Sarah
Hayley
Hannah
Ryan
Sachin


Universe = Class
Boys:
Josh
Ben
Boys
and
Girls
Girls:
Sarah
Jennifer
Boys and
Girls and
Runners
Girls and
Runners:
Hayley
Boys and
Runners:
Ryan
Sachin
Runners

Problems

1.Given the set, name a rule:

a) {1.3.5.7.9}

b) {3,6,9,12,15}

c) {gouda, cheddar, mozzarella, swiss,edam}

d) {buzz, waltz, quiz, chez}

2. Give a set that follows the rule:

a) Numbers divisible by 9

b) Words containing 3 or more vowels

c) Members of your family

3. Determine whether the element is in($\in$) or not in($\notin$) the set:

a) Green {colours in the Canadian flag}

b) Cat {Dog, Fish, Rabbit, Snake, Guinea Pig}

c) 12 {1,1,2,3,5,8,...}

(Hint: research Fibonacci if stuck!)

4. Determine the size of the following sets:

a) |{students in your class}|

b) |{Humans who live on Jupiter}|

c) |{All numbers}|

d) |{number of people in this room}| (Read this carefully!)

5. Are the following sets equal?

a) {1,3,5,9} and {9,1,5,3}

b) {1,3,5,7,9} and {odd numbers between 0 and 10}

c) {1,3,5,7,9} and {1,3,5,7,9,...}

6. Let A = {1,3,5,6,7} and B = {3,5,7}.

a) Is A $\subset$ B?

b) Is B $\subset$ A?

7. Given the universal set $\mathbb{U}$ = {all positive whole numbers less than 12},
P = {even numbers} and M = {numbers that are multiples of 3}, find:

a) $\overline{P}$

b) $\overline{M}$

8. Find the resulting set. Where necessary, describe the set in words.

a) {Numbers divisible by 2} $\cap$ {Odd numbers}

b) {Words that end with a t} $\cup$ {Words that start with with a t}

c) {Countries outside Canada you've visited} $\cup$ {Countries outside Canada you haven't visited}

9. Let A = {Adrian, Brittney, Connor, Dominique, Emily}, B = {Robert, Dominique, Adrian, Hillary, Fran}, C = {George, Brittney, Dominique, Emily, Hillary}, and D = {Connor, Robert, Fran, Adrian, George}.

a) What is A $\cup$ B? A $\cup$ B $\cup$ C? A $\cup$ B $\cup$ C $\cup$ D?

b) What is A $\cap$ B? A $\cap$ C? B $\cap$ C $\cap$ D?

c) What is $\overline{A} \cup \overline{B}$? What is $\overline{(A \cap B)}$?

d) What is $\mathbb{U}$?

e) What is |{(A $\cap$ B)$\cup$ (C $\cap$ D)}|?

f) Draw a Venn diagram for the sets A and B.

g) Draw a Venn diagram for the sets B, C and D.

10. DeMorgan's Laws state that for two sets A and B, the following relations hold:

$\overline{A \cap B} = \overline{A} \cup \overline{B}$ and

$\overline{A \cup B} = \overline{A} \cap \overline{B}$.

Given the universal set {1,2,3,4,5,6,7,8,9,10}, we have the following 4 sets:

A = {1,2,3,4,7}, B = {2,3,5,7,10}, C= {4,6,8,9,10} and D = {1,2,6,8,9}.

Using DeMorgan's Laws, find $\overline{(A \cap B)} \cup \overline{(C \cap D)}$.

Solutions to Problems

1. a) {odd numbers}

b) {Numbers divisible by 3}

c) {Types of cheeses}

d) {Words that end in "z"}

2. a) {9,18,27,36,45...}

b) {telephone, cucumber, mathematics,...}

c) Answers will vary depending on student's family; ex, {Mom, Dad, Sam}

3. a) Green $\notin$ {colours in the Canadian flag}

b) Cat $\in$ {Dog, Fish, Rabbit, Snake, Guinea Pig}

c) 12 $\notin$ {1,1,2,3,5,8...} (The next number is 13 since 8+5 = 13)

4. a) Answer will vary depending on number of students in class
b) 0
c)∞
d)1 (The set containing the number of people in the room is a fixed number (i.e. 40), so the size of that set is just 1)

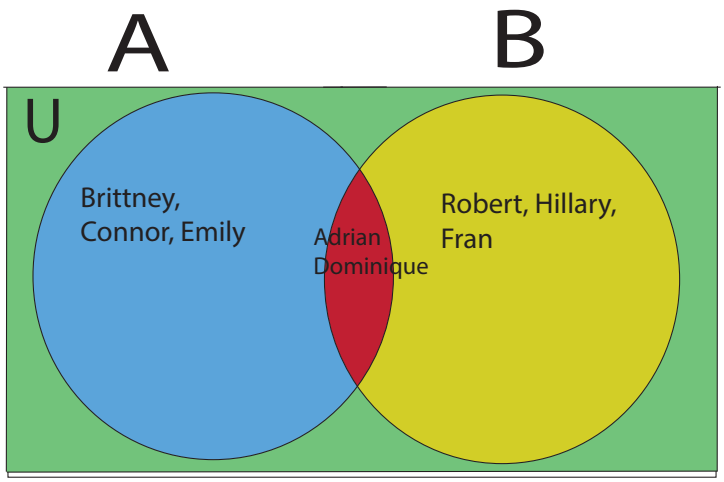
5. a) {1,3,5,9} = {9,1,5,3}
b) {1,3,5,7,9} = {odd numbers between 0 and 10}
c) {1,3,5,7,9} $\neq$ {1,3,5,7,9...}

6. a) A $\not\subset$ B
b) B $\subset$ A

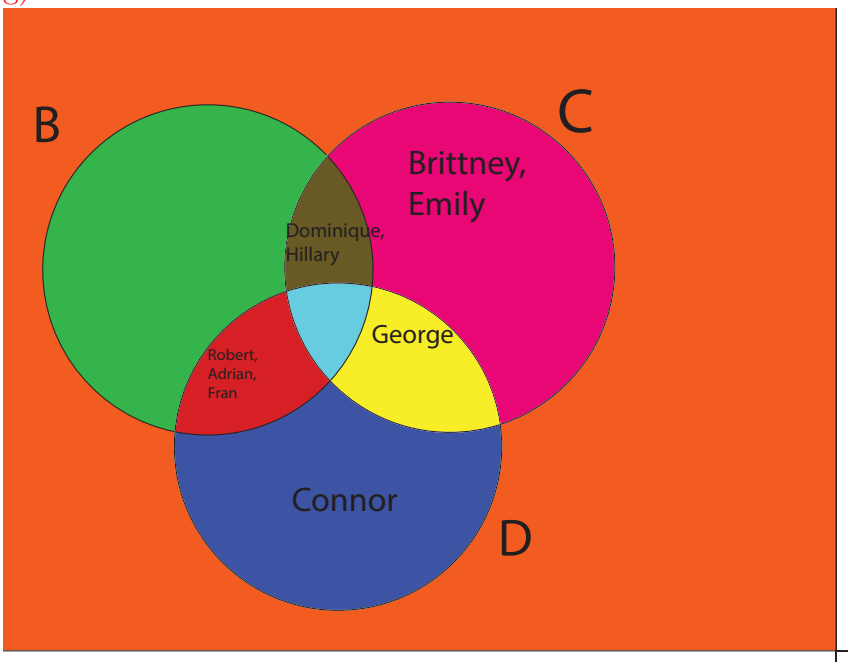
7. a) {1,3,5,7,9,11}
b) {1,2,4,5,7,8,10,11}
8. a) $\emptyset$
b) {The, Train, That, let,...}
c) {All countries}

9. a) A $\cup$ B = {Adrian, Brittney, Connor, Dominique, Emily, Robert, Hillary, Fran}
A $\cup$ B $\cup$ C = {Adrian, Brittney, Connor, Dominique, Emily, Robert, Hillary, Fran, George}
A $\cup$ B $\cup$ C $\cup$ D = {Adrian, Brittney, Connor, Dominique, Emily, Robert, Hillary, Fran, George}
b) A $\cap$ B = {Adrian, Dominique}
A $\cap$ C = {Brittney, Dominique, Emily}
B $\cap$ C $\cap$ D = $\emptyset$
c) $\overline{A} \cup \overline{B}$ = {Fran, George, Hillary, Robert, Brittney, Connor, Emily}
$\overline{(A \cap B)}$ = {Fran, George, Hillary, Robert, Brittney, Connor, Emily}
d) $\mathbb{U}$ = {Adrian, Brittney, Connor, Dominique, Emily, Fran, George, Hillary, Robert}
e)|{(A $\cap$ B)$\cup$ (C $\cap$ D)}| = 3 since
{(A $\cap$ B)$\cup$ (C $\cap$ D)} = {Adrian, Dominique, George}

f)


A
B
U
Brittney,
Connor, Emily
Adrian
Dominique
Robert, Hillary,
Fran

g)


B
C
Brittney,
Emily
Dominique,
Hillary
George
Robert,
Adrian,
Fran
Connor
D

10. We will use the fact that $\overline{(A \cap B)} \cup \overline{(C \cap D)} = (\overline{A} \cup \overline{B}) \cup (\overline{C} \cup \overline{D})$.

$\overline{A} = \{5,6,8,9,10\}$

$\overline{B} = \{1,4,6,8,9\}$

$\overline{A} \cup \overline{B} = \{1,4,6,8,9,10\}$

$\overline{C} = \{1,2,3,5,7\}$

$\overline{D} = \{3,4,5,7,10\}$

$\overline{C} \cup \overline{D} = \{1,2,3,4,5,7,10\}$

So $(\overline{A} \cup \overline{B}) \cup (\overline{C} \cup \overline{D}) = \{1,2,3,4,5,6,7,8,9,10\}$. which is the universal set.