MATH CIRCLES FEBRUARY 25 - PROBLEMS

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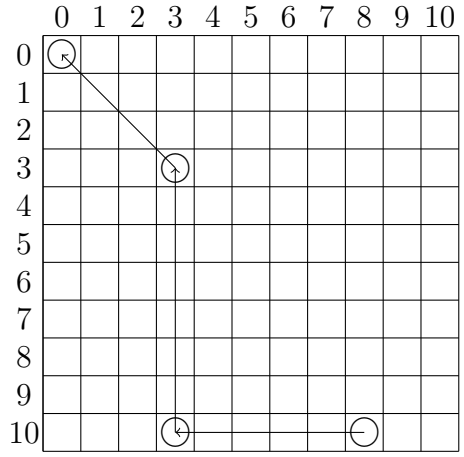
The first game has two players, Player 1 and Player 2. There is a pile of stones, and a legal move is to remove at least one, and at most two stones. The player that takes the last stone wins.

(1) Who has a winning strategy when the number of stones is:
   (a) 15
   (b) 20
   (c) 10
   (d) 600 (maybe don't do this one by experimentation)
(2) Who has the winning strategy (and what is it) for an arbitrary number of stones?
(3) If we change the game so that players may remove 1,2, or 3 stones, what is the winning strategy if the number of stones is
   (a) 20
   (b) 25
   (c) 30
   (d) 35
(4) What happens in general?
(5) What about if the players are allowed to remove $1, 2, 3, 4, 5, 6, 7, 8, 9$, or 10 stones and there are 599 at the start?

The next game played on a rectangular grid (think Chess board) with a Queen. The players alternate moving the Queen, and the player that manages to put her on the top left cell wins. The catch is that the players may not move her to the right or down. Only left, up, and diagonally up-left. Here is an example of a sequence of legal moves.



(6) Try this game with a friend and see if you can figure out who has the winning strategy for a few starting positions. Start close to the $(0, 0)$ block.

(7) Try to map out a winning strategy by identifying squares that you want to get to, and squares that you want to avoid.

(8) Here is another game: There are two piles of stones, and the legal moves are to remove stones from one of the two piles, or remove the same number of stones from each pile. At least one stone must be removed per turn, and two players alternate making moves. The player to remove the last stone wins. Who has the winning strategy for this game, and what is it?