UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles

November 24/25, 2015

Review

What have you learned in the past seven weeks?

First and foremost, you now know of several areas of study in mathematics in which mathematicians today are interested in developing. Second, you learned a bit about these topics. Many of these subjects require years of study and practice to understand them. We chose these topics because they are areas of interest for us but are also easy enough to explain the basics and pick up some useful ideas quickly.

For instance, in the first week you learned about Greek constructions. Although this is not a topic in much use today, the mathematicians that did use it to create new mathematics have become famous names in the mathematics community. For instance, Carl Friedrich Gauss studied constructions and is widely believed to be one of the greatest mathematicians of all time. The logic of how constructions work is useful and is primarily why it is still studied today.

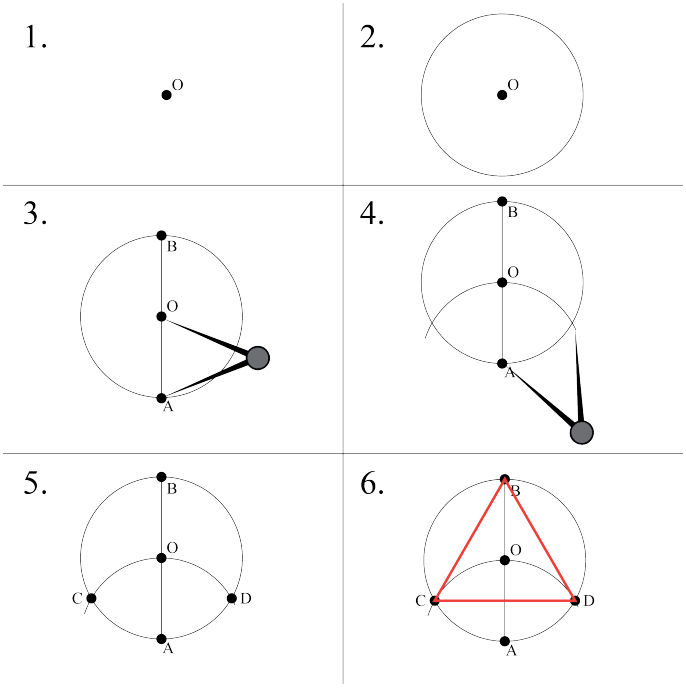
Some of you already know what you want to do for a living. Some of you still have yet to decide. I decided I wanted to become a mathematician at your age (in grade 8) and I am still happy with that decision. Mathematics is a great subject and no matter what you choose to study you should use the analytical skills and tools you learn in math classes throughout your life. They can be incredibly useful.

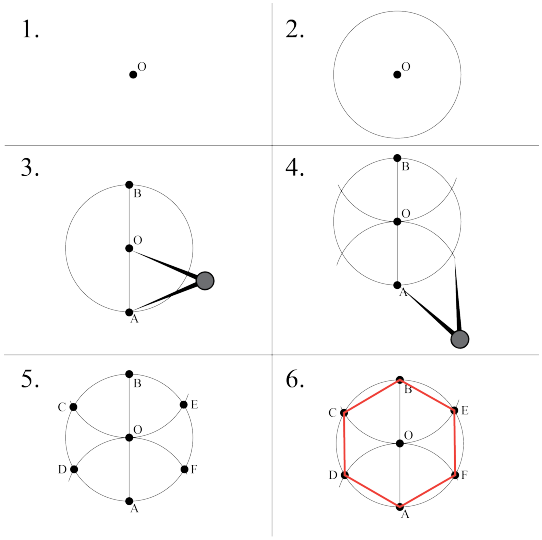
This lesson is designed as a contest. The person or team that can score as many points as possible by correctly completing problems wins. Here is a scoring chart for an individual or a team. Each topic is a separate question, so question 1 is Greek Constructions and question 5 is Tessellations, for example.

Question:	1	2	3	4	5	6	7	Total
Score:								
Out of:	/2	/4	/5	/15	/2	/5	/7	/40

1. Greek Constructions (2 points)

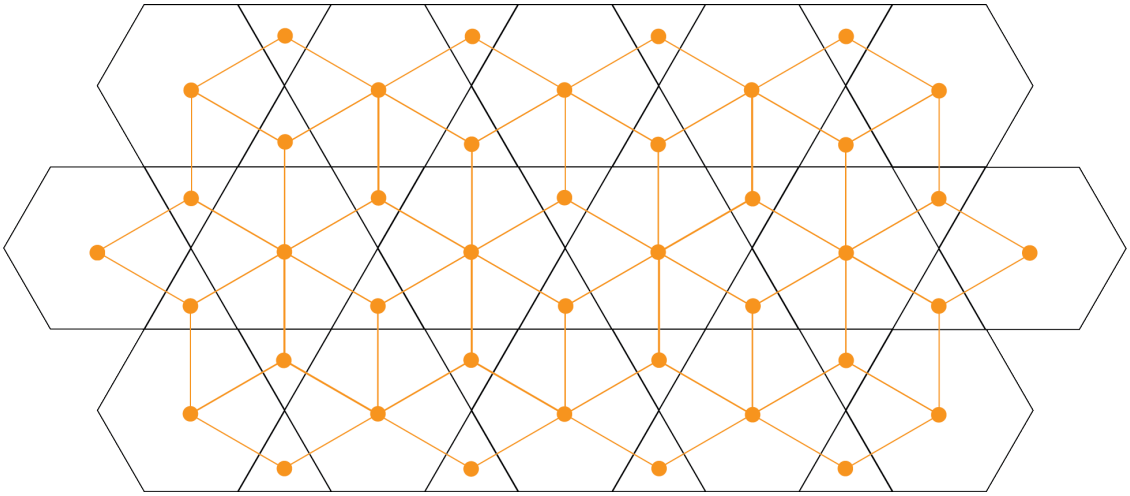
Follows these instructions to draw an equilateral triangle and then a regular hexagon:





2. Graph Theory (4 points)

What is the smallest number of colours you need to colour this tessellation? A graph is drawn on top of the tessellation to help. How do you know it can be coloured in with that many colours?



3. Computer Algorithms (3 + 2 points)

1. Here are three lists of numbers. Sort each of them using a different sorting algorithm (pick from insertion, selection, and merge sort):

   - $5, 3, 6, 1, 5, 6, 8, 10, 3, 1$
   - $1, 2, 7, 3, 8, 3, 1, 5, 10, 9$
   - $10, 2, 0, 1, 9, 9, 9, 2, 3, 5$

2. Determine the algorithm in the following two patterns:

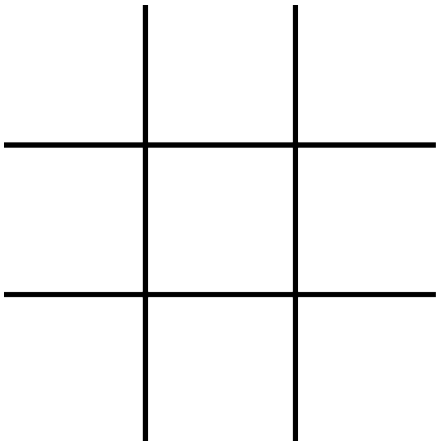
Four is the Cosmic Number 1 is 3, 3 is 5, 5 is 4, 4 is 4, and 4 is the cosmic number. 12 is 6, 6 is 3, 3 is 5, 5 is 4, 4 is 4, and 4 is the cosmic number. 50 is 5, 5 is 4, 4 is 4, and 4 is the cosmic number.	One is the Only Number 3 is 0, 0 is 1, 1 is 1, and 1 is the only number. 1000 is 2, 2 is 1, 1 is 1, and 1 is the only number. 72 is 1, 1 is 1, and 1 is the only number.

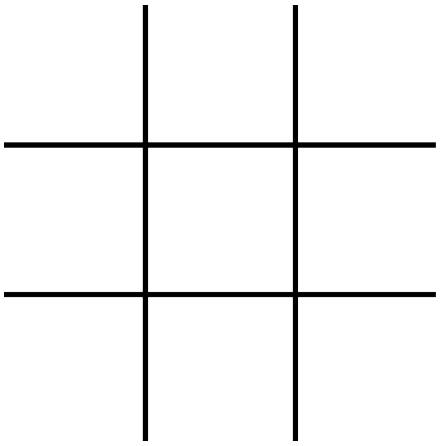
Hint: It helps to write the numbers out as words.

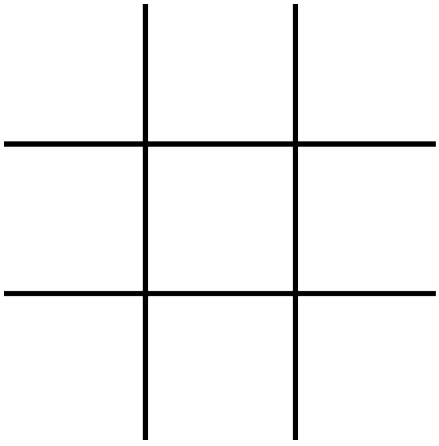
4. Game Theory (0 points for loss, 1 for tie, 3 for win)

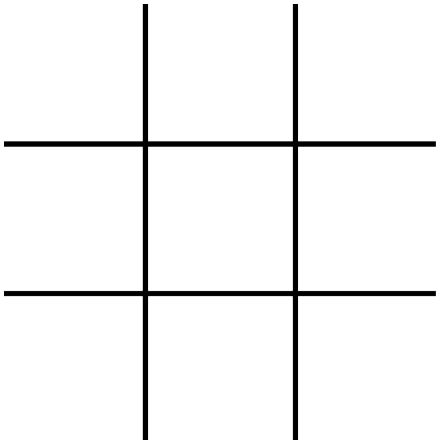
Tic Tac Toe is a simple game to understand, play, and play so well that the only outcome will be a tie. Find your optimal strategy to never lose.

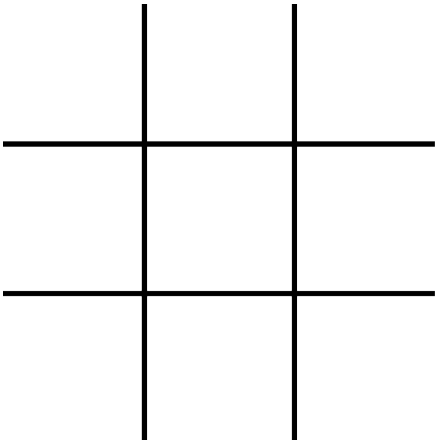
Play five games of tic tac toe against any opponent. If you lose, you will get 0 points, a tie earns you 1 point, and a win earns you 3 points.



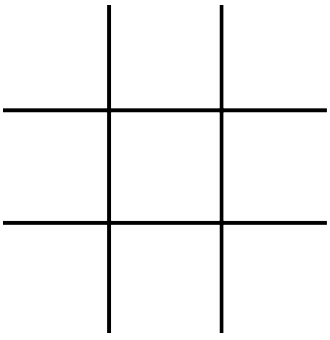






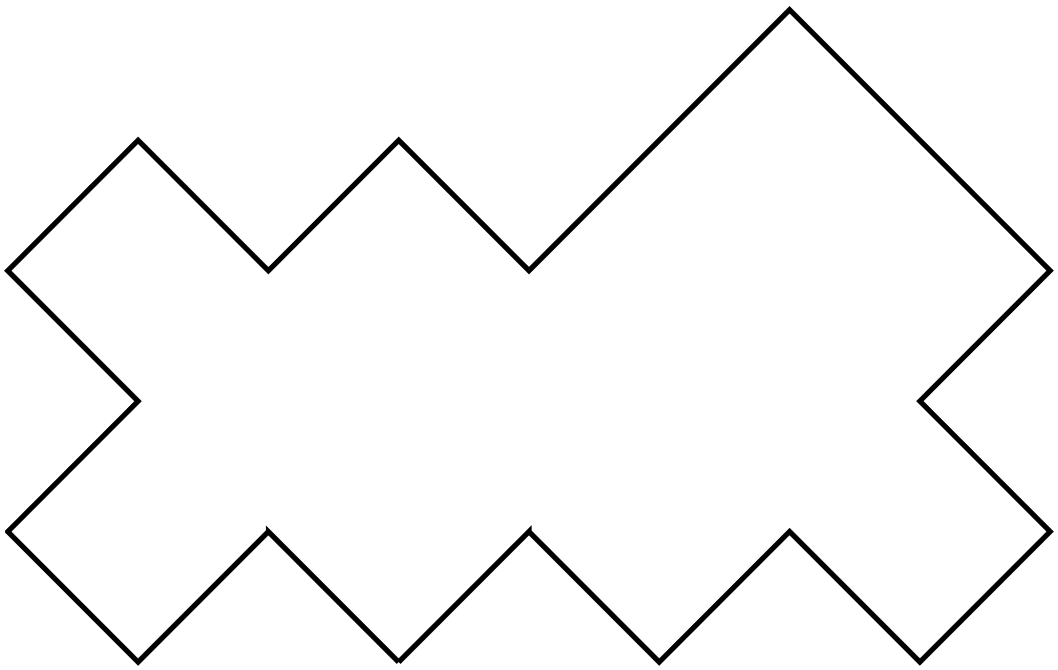


Extra:



5. Tessellations (2 points)

On the following image, draw one line (does not have to be straight) that cuts the image into two **identical** shapes.



6. Logic (2 + 1 + 1 + 1 points)

1. As punishment for throwing pizza on the roof of your house, you have been made to do your family's laundry for the week. This is the first time you have ever done laundry and while folding your own clothes, you forget to pair matching socks together. You own 10 identical pairs of white socks and 10 identical pairs of black socks. After stuffing them in your sock drawer before Math Circles, the power shuts off and you are left in complete darkness. You still have to go to Math Circles tonight so you grab some number of socks before you leave. What is the smallest number of socks you need to pull from your drawer of unpaired socks to ensure you have two of the same colour?

2. Determine the value of the following statements:

   (a) $(True \wedge False) \vee True =$

   (b) $(True \vee False \vee False \vee True \vee False) =$

   (c) $\neg((False \wedge True) \veebar True) =$

7. Knot Theory (1 + 1 + 2 + 3 points)

1. Draw a knot with a minimal crossing number of zero.

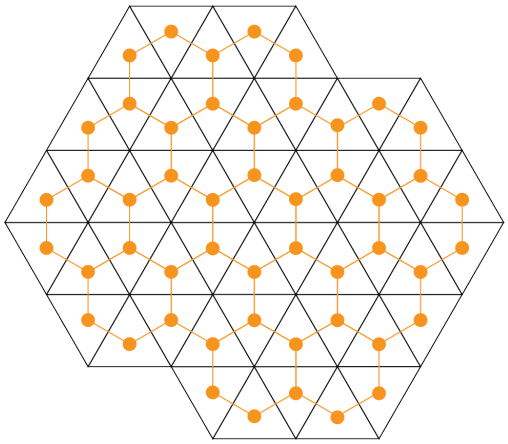
2. Draw a knot with a minimal crossing number of three.

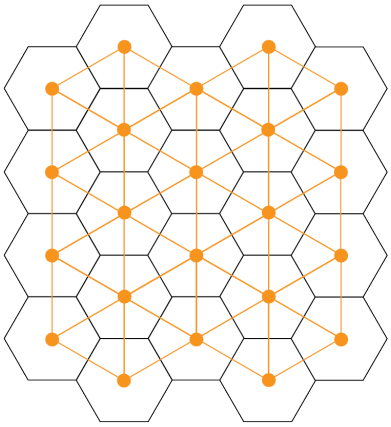
3. * Draw a knot with a minimal crossing number of four.

4. Why are there no knots with minimal crossing numbers of one or two?

Exercises:

1. Here are two more tessellations with related graphs. Determine the fewest number of colours that are needed to colour each tessellation/graph.





2. Brien is a programming a computer to win a game. He wants to design an algorithm to guess a number from 1 to 300 that you pick. To win the game, Brien must guess the number in under 10 tries. Every time the algorithm guesses a number, you must tell it if its guess was too high or too low. You cannot lie about your number.

   (a) Brien's first algorithm guesses the numbers from 1 to 300 until its guess is correct. Will this algorithm win the game? Can you think of a better way to guess?

   (b) Brien's second algorithm is called a binary search algorithm. Find a partner and play the guessing game with them. Use Brien's algorithm:

   - Always guess the median (middle number) of the remaining set of numbers.

   Does Brien's new algorithm win the game for you? * Will it always win the game?

3. The University of Waterloo is deciding to renovate their Mathematics and Computer building (MC). The designers ask the Math Circles for inspiration. They want to tile the floors in MC with a single shape (such as a square) but they do not want it to be as boring as a regular triangular, square, or regular hexagonal tiling. In other words, they are looking for an irregular shape to tile the floors of MC. **What irregular shape do you suggest they use?** The designers will pick the most impressive tile but still want a tile that has straight and simple edges so it is cheap to produce.

4. Brussel Sprouts is a game for two players. Here is how you play:

   (a) Begin with a set of crosses (an X) spread across the paper.

   (b) On each turn, a player draws a line between two crosses (a cross has four free ends). On that line, the player adds a slash in the middle (two new ends).

      i. The new line cannot touch or cross itself or any other line.

      ii. Ends are always being connected as lines are drawn.

   (c) The first player who cannot make a move loses the game.

   Play five games, each time alternating who goes first. Play with two crosses to start, then increase the number of crosses for each game.

5. This question is from the 2011 Gauss contest produced by the CEMC.

   In the addition of the three-digit numbers shown, the letters A, B, C, D, and E each represent a single digit.

$$\begin{array}{cccc} & A & B & E \\ & A & C & E \\ + & A & D & E \\ \hline 2 & 0 & 1 & 1 \end{array}$$

   The value of $A + B + C + D + E$ is:

   (a) 34 (b) 21 (c) 32 (d) 27 (e) 24

6. From the same contest:

   Three pumpkins are weighed two at a time in all possible ways. The weights of the pairs of pumpkins are 12 kg, 13 kg, and 15 kg. How much does the lightest pumpkin weigh?

   (a) 4 kg (b) 5 kg (c) 6 kg (d) 7 kg (e) 8 kg

7. Draw a knot with an unkotting number of one.