Intermediate Math Circles
February 17, 2016
Contest Prep: Number Theory

Part 1: Prime Factorization

A **prime number** is ____

An integer greater than 1 is **composite**, if ____________________________

How do we check if a number $n > 1$ is prime?

Divisibility Tests:

Divisibility by 2:

Divisibility by 3:

Divisibility by 5:

Divisibility by 11:

(Although the following divisors are not prime divisors, these tests can be helpful.)

Divisibility by 4:

Divisibility by 8:

Divisibility by 9:

Examples: Are the following numbers prime? If they are composite, express them as a product.

1. 151

2. 517

3. 273

4. 293

The Fundamental Theorem of Arithmetic:

Examples: Write each number as a product of prime factors.

1. 517

2. 273

3. 792

Example: (1999 Pascal #22) If

$$w = 2^{129} \times 3^{81} \times 5^{128},$$
$$x = 2^{127} \times 3^{81} \times 5^{128},$$
$$y = 2^{126} \times 3^{82} \times 5^{128}, \text{ and}$$
$$z = 2^{125} \times 3^{82} \times 5^{129},$$

then write w, x, y and z in order from smallest to largest.

Example: (Cayley 1996 #20) Determine the smallest perfect square greater than 4000 that is divisible by 392.

Problem Set 1

1. How many positive divisors, other than 1 and the number itself, does 23 400 have?
2. The product of 20^{50} and 50^{20} is written as an integer in expanded form. What is the number of zeros at the end of the resulting integer?
3. The integer 636 405 may be written as the product of three 2-digit positive integers. What are these three integers?
4. What is the smallest positive integer whose digits have a product of 2700?

Part 2: Solving Problems Involving Digits

Decimal Expansion

Most often in mathematics we use the **decimal system** of numbers. The digits of a number are multiplied by a power of 10 depending on their position. We say that we are using **base 10**.

For example the decimal expansion of 4327 is 4327 = ______________________________

In solving some problems that make reference to the digits of numbers we can use a general decimal expansion.

For example, a three digit number can be represented by ______________________________

Example: Show that the difference between any three-digit number, in which no digits are 0 or equal, and the number formed by reversing the digits is always a multiple of 99.

Example: (Pascal 2003) The people of Evenland never use odd digits. Instead of counting 1, 2, 3, 4, 5, 6, an Evenlander counts 2, 4, 6, 8, 20, 22. What is an Evenlander's version of the integer 111?

Example: The five-digit number 9T67U, where T and U are single digits, is divisible by 36. Determine all possible values of T and U.

Problem Set 2

1. How many two-digit positive integers are increased by 11 when the order of the digits is reversed?

2. A six-digit number is formed by repeating a three-digit number; for example 123 123 or 265 265. What is the largest common factor of all such numbers?

3. The three-digit number $2A4$ is added to 329 and gives $5B3$. If $5B3$ is divisible by 3, then what is the largest possible value of A?

4. A four-digit number which is a perfect square is created by writing Anne's age in years followed by Tom's age in years. Similarly, in 31 years, their ages in the same order will again form a four-digit perfect square. Determine the present ages of Anne and Tom.