Intermediate Math Circles
Wednesday March 02, 2016
Introduction to Vectors II

1. If $\vec{u} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, find the following:

   (a) i. $\vec{u} \cdot \vec{v}$

   ii. $\vec{v} \cdot \vec{u}$

   iii. $\hat{u}$

   iv. $(\vec{u} + \vec{v}) \cdot \vec{w}$

   v. $(3\vec{u}) \cdot \vec{v}$

   vi. $3(\vec{u} \cdot \vec{v})$

   vii. $\vec{w} \cdot \vec{w}$

   viii. $\vec{u} \times \vec{v}$

   ix. $\vec{v} \times \vec{w}$

   x. $\vec{w} \times \vec{u}$

   (b) Find $d(\vec{u}, \vec{v})$, $d(\vec{v}, \vec{w})$, and $d(\vec{w}, \vec{u})$.

   (c) Find the area of the triangle formed by $\vec{u}$ and $\vec{v}$ using the formula

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \, \|\vec{v}\| \sqrt{1 - (\hat{u} \cdot \hat{v})^2}$$

2. Let $\vec{x} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} k \\ 4 \end{bmatrix}$, what value of k makes these vectors orthogonal?

3. Let $\vec{x} = \begin{bmatrix} 3k+2 \\ -3 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$, what value of k makes these vectors orthogonal?

4. Recall that $\|\vec{z}\|^2 = \vec{z} \cdot \vec{z}$. Prove that if $\vec{u} \cdot \vec{v} = 0$, then $\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$.

5. Prove $\vec{u} \cdot \vec{u} \geq 0$ and $\vec{u} \cdot \vec{u} = 0$ only if $\vec{u} = \vec{0}$.

Harder Given that $|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \, \|\vec{v}\|$, use the properties of dot products to prove

$$\|\vec{u} + \vec{v}\|^2 \leq (\|\vec{u}\| + \|\vec{v}\|)^2$$

This property is called the Minkowski Inequality and can be simplified by taking the square root of both sides.

Sudoku to end the day

	1		5	6				3
	7		1					9
5				3		1	6	
		9	7				1	
	4						9	
	5				3	7		
	3	7		1				5
2					5		3	
1				2	4		7	