Intermediate Math Circles
Wednesday March 09, 2016
Introduction to Vectors III

1. In the diagram, A is the midpoint of OP and N is the midpoint of PQ, with $\overrightarrow{OB} = \frac{1}{4}\overrightarrow{OQ}$. Let $\vec{u} = \overrightarrow{OA}$ and $\vec{v} = \overrightarrow{OB}$. Write the following vectors in terms of $\vec{u}$ and $\vec{v}$.

   (a) $\overrightarrow{AP}$

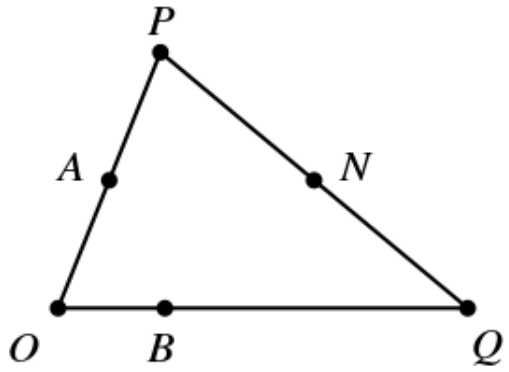
   (b) $\overrightarrow{QO}$

   (c) $\overrightarrow{PB}$

   (d) $\overrightarrow{QP}$

   (e) $\overrightarrow{NQ}$

   (f) $\overrightarrow{ON}$



2. What value of a makes $\begin{bmatrix} -8 \\ a \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \\ a \end{bmatrix}$ orthogonal?

3. Let $\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$. If $\vec{z} = \vec{u} \times \vec{v} = \begin{bmatrix} 2 \\ 16 \\ 11 \end{bmatrix}$, what is $\vec{u} \bullet \vec{z}$? What is $\vec{v} \bullet \vec{z}$? What is special about the vector created by the cross product of two vectors?

4. Consider the points $P(1, 5, 7)$, $Q(2, 4, 3)$, and $R(3, 3, -1)$. What is the relationship between $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$?

5. Find the vector equation for the line that passes through $P(-5, 2, 10)$ and $Q(3, -4, -4)$.

6. Find the distance between the points $P(2, 3, 0)$ and $Q(-1, 2, 4)$.

7. Let $\vec{u} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. Find the following:

   (a) $\vec{u} + \vec{v}$

   (b) $\frac{3}{2}\vec{u}$

   (c) $\vec{u} \bullet \vec{v}$

(d) $d(\vec{u}, \vec{v})$

(e) $\|\vec{u}\| + \|\vec{v}\|$

(f) $\hat{u}$ and $\hat{v}$

(g) $\vec{u} \times \vec{v} = \begin{bmatrix} u_2v_3 - u_1v_2 \\ u_3v_1 - u_1v_3 \\ u_1v_2 - u_2v_1 \end{bmatrix}$

(h) $\|\vec{u} \times \vec{v}\|$

8. (Harder) Let $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$.

   (a) Find $\vec{u} \bullet (\vec{v} \times \vec{w})$. This is the volume of the 3D parallelogram (known as a parallelepiped) created by the three vectors.

   (b) Find $\|\vec{v} \times \vec{w}\|$ given the formula

$$\|\vec{v} \times \vec{w}\|^2 = (\vec{u} \bullet \vec{u})(\vec{v} \bullet \vec{v}) - (\vec{u} \bullet \vec{v})^2$$

Sudoku to end the day

1		7		8	9			
		6		3			9	
				4				7
	2		4					1
	4			9			7	
3					8		4	
2				1				
	6			7		4		
			6	2		8		3