



# Intermediate Math Circles

## Wednesday November 23 2016

### Problem Set 8

- The point of this first exercise is to get warmed up and recall what we learned last time.
  - Find the remainder when 3434 is divided by 17.
  - What is the tens digit of  $(99)^{881}$ ?
  - January 1, 2017 is on a Sunday. What day of the week will February 14, 2018 be on?
- Calculate the remainder when  $(17)^{125}$  is divided by 5. (Hint: Use Fermat's Little Theorem)
- True or False: Every number of the form  $a^2 - a$  is even.
- What is  $15002^{(3^4)}$  congruent to modulo 3? (i.e. 0, 1, or 2?)
- Show that the check digit *always* detects an error made when making exactly one typo in a UPC.
- Which of the following UPC's have correct check digits?



- What is the check digit of the UPC (0, 3, 0, 9, 5, 5, 1, 6, 9, 8, 2, \*)? What do you notice about this answer when compared to the above question?
- Determine when the check digit of a UPC does *not* detect an error made by switching two adjacent numbers.
- In Florida, the fourth and fifth digits from the end of a driver's license number give the year of birth. The last three digits for a male with birth month  $m$  and birth date  $b$  are represented by  $40(m - 1) + b$ . For females the digits are  $40(m - 1) + b + 500$ . Determine the dates of birth of people who have last five digits 42218 and 53953.
- For driver's license numbers issued in New York prior to September of 1992, the three digits preceding the last two of the number of a male with birth month  $m$  and birthdate  $b$  are represented by  $63m + 2b$ . For females the digits are  $63m + 2b + 1$ . Determine the dates of birth and genders which correspond to the numbers 248 and 601.
- The state of Utah appends a ninth digit  $a_9$  to an eight-digit driver's license number  $a_1a_2 \cdots a_8$  so that  $(a_1, a_2, \dots, a_9) \cdot (9, 8, 7, 6, 5, 4, 3, 2, 1) \equiv 0 \pmod{10}$ . If you know that the license number 149105267 has exactly one digit incorrect, explain why the error cannot be in position 2, 4, 6, or 8.