



Intermediate Math Circles

Wednesday November 2 2016

Problem Set 5

Here are the November 2 exercises. (6), (7), (8), and (9) are from October 26.

1. Find the binary expansions of 3, 7, 10, 15, 21, 30, 32, and 53.
2. Find the binary expansions of 6, 14, 20, 30, 42, 60, 64, and 106. Compare your answers to the answers in the previous problem. Explain your observation. [Hint: You don't need to do all of them to notice the pattern. Start with the small ones.]
3. Pick two 2-digit numbers and write them in binary. Add and multiply them in binary using the regular method for addition and multiplication. Keep in mind that $1 + 1 = 10$ and $1 + 1 + 1 = 11$ in binary. Convert the answers back to decimal and check to see if you were correct. Try another pair of numbers if you like! You don't have to stick to two digit numbers.
4. Recall that a number is divisible by 3 exactly when the sum of its digits is divisible by 3. For example, since $2 + 7 + 9 + 1 + 5 = 24$, which is a multiple of 3, the number 27915 is a multiple of 3.
 - (a) Find an example to show that this does not work for binary expansions.
 - (b) Using inspiration from the "divisibility by 11" trick, find a trick that will tell you whether or not a number, written in binary, is divisible by 3.
5. For each Nim state, determine whether or not it is a next or previous player win. For those that are next player win, give a correct next move. [Hint: Use the first exercise.]
 - (a) $53 \oplus 32 \oplus 30 \oplus 15$
 - (b) $15 \oplus 7 \oplus 3$
 - (c) $30 \oplus 21 \oplus 10$
6. Find the Zackendorf decompositions of 10, 15, 20, 500, 610, and 1000.
7. Play Fibonacci Nim for a few minutes. Try to find a winning strategy for a few small examples. Can you find a winning strategy in general? Warning: This one is hard. The general strategy involves the Zackendorf decomposition of the number.
8. Explain why no two numbers in a Zackendorf decomposition are consecutive Fibonacci numbers. Here consecutive means they appear next to one another in the Fibonacci sequence.
9. Explain why the same number can not occur twice in the Zackendorf decomposition of a number.
10. Find a strategy for Kayles.



11. Show that Player 1 has a winning strategy in Chomp in the case that the game starts with a rectangular grid. [Hint: Nobody knows a winning strategy, but you can show that there is one. If Player 2 had a winning strategy, how would they respond to Player 1 removing just the top right cell?]
12. No general winning strategy for Player 1 is known for a rectangular grid. However, you can find one for some special cases. Find a winning strategy for the Player 1 in the following situations:
 - (a) A square grid.
 - (b) A $2 \times n$ grid.