



Grade 6 Math Circles

November 15th/16th
Arithmetic Tricks

We are introduced early on how to add, subtract, multiply, and divide. As we learned more math, we had to deal with bigger numbers requiring more time to compute the answer. In this week's math circles we will learn some tricks to speed up our arithmetic skills.

Being able to calculate quickly alleviates the load on your limited working memory, allowing your brain to concentrate on higher order thinking and problem solving. As you progress through school, you will be solving more and more difficult problems, you do not want to be boggled down over calculations. Not to mention, it is pretty impressive to show others your affinity for arithmetic!

However, some of these arithmetic tricks may seem unnecessary at first or even overly complicated. This may seem true initially, but mastery of these tricks will allow you to perform calculations in your head pretty quickly. This comes with a lot of practice. The standard methods you have learned in school may be too cumbersome for you to perform in your head. The goal eventually is to be able to perform all these tricks in your head without writing anything down.

Multiplying by 5

Example. Evaluate 137×5 .

A clever trick is to realize that $5 = 10 \div 2$.

$$\begin{aligned} 137 \times 5 &= 137 \times 10 \div 2 \\ &= 1370 \div 2 \\ &= 635 \end{aligned}$$

This is much easier than multiplying by 5 head on.

Rule: When multiplying by 5, multiply by 10 and divide by 2.

Exercise. Evaluate the following

1. 15×5

4. 65×5

2. 28×5

5. 18×5

3. 90×5

6. 22×5

Dividing by 5

Rule: Dividing by 5 is the same as multiplying by 2, then dividing by 10.

Example. $325 \div 5$

$$\begin{aligned} 325 \div 5 &= 325 \times 2 \div 10 \\ &= 650 \div 10 \\ &= 65 \end{aligned}$$

Why does this work? Recall that $325 \div 5 = \frac{325}{5}$

$$\begin{aligned} \frac{325}{5} &= 325 \times \frac{1}{5} \\ &= 325 \times \frac{2}{10} \end{aligned}$$

We can see in the final step that multiplying by $\frac{2}{10}$ is the same as multiply by 2 and then dividing by 10.

Exercise. Evaluate the following

1. $85 \div 5$

5. $440 \div 5$

2. $135 \div 5$

6. $360 \div 5$

3. $275 \div 5$

7. $95 \div 5$

4. $300 \div 5$

Multiplication of 11 With Any 2 Digit Number

To multiply a 2 digit number with 11.

1. Write down the **first digit** of the non-11 number. This is going to be our **first digit** .
2. Write down the **second digit** of the non-11 number. This will be our **last digit**.
3. Add the two digits and place the sum between the previous two digits. This is the **middle digit**.
4. If the sum is greater than 10, carry over the tens digit if necessary from the middle digit to the first digit.

Example. 25×11

The non-11 term is 25.

1. The **first digit** of 25 is 2.
2 will be the **first digit** of the product of 25×11 .

2

2. The **second digit** of 25 is 5.

5 will be the **last digit** of the product of 25×11

$$\begin{array}{ccc} 2 & & 5 \\ \hline \end{array}$$

3. $2 + 5 = 7$ and place it between 2 and 5.

7 will be the middle digit of the product of 25×11

$$\begin{array}{ccc} & 2 + 5 & \\ 2 & 7 & 5 \\ \hline \end{array}$$

$$\therefore 25 \times 11 = 275$$

Example. 38×11

In this example, we have to carry over the sum. Since 38 is the non-11 term, we write down 3 as the first digit and 8 as the last digit.

$$\begin{array}{ccc} 3 & & 8 \\ \hline \end{array}$$

All that remains is to find the middle digit. Adding the two digits of 38, $3 + 8 = 11$. Since 11 is greater than 9, we have to carry over the 1 and add it to 3.

$$\begin{array}{ccc} & 3 + 8 & \\ 3 & 11 & 8 \\ \hline \end{array}$$

We can only have a single digit in the middle, so we have to carry over the 1 over

$$\begin{array}{ccc} 3 + 1 & 1 & 8 \\ \hline \end{array}$$

We carry over by adding the 1 to the digit to the right of it which is 3

$$\begin{array}{ccc} 4 & 1 & 8 \\ \hline \end{array}$$

$$\therefore 38 \times 11 = 418$$

Exercise. Evaluate the following

1. 45×11

4. 85×11

2. 12×11

5. 94×11

3. 67×11

6. 11×11

Multiplying Any Number by 11

To multiply any number by 11,

1. Write down the number's right most digit
2. Add the that digit to the digit on the left, write down the units digit, and carry over the tens digit if it is greater than 1
3. Proceed to the next digit and repeat the process all over
4. Once you reach the last digit, write down the last digit

Example. Evaluate 54321×11

$$5432111 = 5(5 + 4)(4 + 3)(3 + 2)(2 + 1)1 = 597531$$

Example. Evaluate 62473×11

$$\begin{aligned} 62473 \times 11 &= 6(6 + 2)(2 + 4)(4 + 7)(7 + 3)3 \\ &= 686(4 + 7)(7 + 3)3 \\ &= 6871(7 + 3)3 \\ &= 687203 \end{aligned}$$

Exercise. Evaluate the following

1. 111×11

4. $4,389 \times 11$

2. 345×11

5. 72831×11

3. $2,359 \times 11$

6. 9527136×11

Multiplication with 9, 99, 999 ...

Multiplying numbers with 9, 99, or 999 can seem like a hassle but we can actually multiply things take advantage of the **distributive** property of multiplication.

Example. Evaluate 44×9 We know that $9 = 10 - 1$

$$\begin{aligned} 44 \times 9 &= 44 \times (10 - 1) \\ &= 44 \times 10 - 44 \\ &= 440 - 44 \\ &= 440 - 40 - 4 \\ &= 396 \end{aligned}$$

This trick may seem a little pointless, but it becomes increasingly invaluable as we deal with bigger numbers.

Example. Evaluate 68×99

$$\begin{aligned} 68 \times 99 &= 68 \times (100 - 1) \\ &= 68 \times 100 - 68 \\ &= 6800 - 68 \\ &= 6800 - 60 - 8 \\ &= 6732 \end{aligned}$$

Exercise. Evaluate the following

1. 18×9

5. 13×99

2. 45×9

6. 112×99

3. 93×99

7. 178×999

4. 78×99

8. 24×999

Squaring Numbers Ending in 5.

It is also very easy to square any number that ends in 5.

1. First multiply the first digit with the number one bigger than it. i.e if the first digit is 7, then we multiply it by the number one bigger than it so 8.
2. Then place 25 behind their product

Example. Evaluate 65^2

1. The first digit is 6, so we multiply $6 \times 7 = 42$.
2. Then place 25 behind 42. So we get 4225

$$\therefore 65^2 = 4225$$

Squaring numbers that end in 5 but are larger than 100 still works, only now we square the digits in front of the 5.

Example. Evaluate 115^2 .

1. The first few digits in front of 5 is 11, so we multiply 11 with the number one bigger than it, $12 \times 11 = 132$

2. We place 25 behind 132.

$$\therefore 115^2 = 13225$$

Optional: Why does this Work? Suppose we are squaring a number, m that ends in 5. We can express m as a multiple of 10 plus 5 $m = 10n + 5$. Notice carefully that n will be the first digit of our number. For example, $65 = 6 \times 10 + 5$, then $n = 6$ If we square m

$$\begin{aligned} m^2 &= (10n + 5)^2 \\ &= (10n + 5)(10n + 5) \\ &= 100n^2 + 25 + 50n + 50n \\ &= 100n^2 + 100n + 25 &= 100n(n + 1) + 25 \end{aligned}$$

In the expression $100n(n + 1) + 25$, notice that $100n(n + 1)$ is a multiple of 100 and hence the last two digits end are 0 but we are we are adding 25, so now we know that the last two digits must be 25. In addition, the multiple of 100 is $n(n + 1)$ which is the product of n and one above it.

Exercise. Evaluate the following

1. 35^2

5. 195^2

2. 45^2

6. 125^2

3. 75^2

7. 25^2

4. 105^2

Squaring Two digit Numbers

There are actually many strategies for being able to squaring two digit numbers very quickly in your head. The one that we learn today involves an algebraic identity.

To square any number,

1. **Round** the number **down** to the nearest 10. This will be our **first** number.
2. Using the number we are squaring, break the number as a sum of the first number and whatever is left over. Whatever left over is our **second number**
3. Now we use the first and second number to create three different numbers we can add easily in our head!
 - (a) **Square** the **first** number
 - (b) **Square** the **second** number
 - (c) Multiply the first and second number together and then **DOUBLE** it!
4. Add those three numbers and we have the answer.

Example. Evaluate 52^2

1. Rounding 52 down to the nearest 10 gives us 50
2. We can break 52 into a sum of 50 and 2 as what is left over

We will use 52 and 2 to create three numbers we can add easily in our heads!

- (a) Square 50, $50^2 = 2500$
 - (b) Square 2, $2^2 = 4$
 - (c) Find the product of $2 \times 2 \times 50 = 200$
3. Add the 3 numbers $2500 + 200 + 4 = 2704$ and 2704 is our final answer.

Example. Evaluate 67^2

1. Rounding 67 down to nearest 10 gives us 60
2. We can break 67 into a sum of 60 and 7

We will use 60 and 7 to create three numbers we can add easily in our heads!

- (a) Square $60^2 = 3600$
- (b) Square $7^2 = 49$

(c) Find the product of $2 \times 60 \times 7 = 880$

3. Add the three numbers $3600 + 49 + 880 = 4489$

Optional: Why does this work?

We will take advantage of this algebraic identity:

$$(a + b)^2 = a^2 + 2ab + b^2$$

Proof of this identity:

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a^2 + b^2 + ab + ab \\ &= a^2 + 2ab + b^2\end{aligned}$$

We take advantage of that identity to break down a square into easy numbers we can square in our heads. Using the first example 52^2 .

$$\begin{aligned}52^2 &= (50 + 2)^2 \\ &= 50^2 + 2 \times 50 \times 2 + 2^2 \\ &= 2500 + 200 + 4 \\ &= 2704\end{aligned}$$

Exercise. Evaluate the following

1. 17^2

5. 95^2

2. 27^2

6. 76^2

3. 53^2

7. 105^2

4. 49^2

There is an alternate (but nearly identical way) to calculate square roots quickly in your head. Instead of breaking down a number into a sum, we can break it down into a difference.

To square any number,

1. Round the number we are squaring up to the nearest 10. This will be our **first** number.
2. Express that number as a difference of the first number and whatever is left over. Whatever is left over will be our **second** number
3.
 - (a) **Square** the **first** number
 - (b) **Square** the **second** number
 - (c) Multiply the first and second number and then **DOUBLE** it!
4. Add the first two but subtract the last one

Example. This example has already been done, but is to illustrate how we can arrive at the same answer in a slightly different manner.

1. The smallest number divisible by 10 greater than 67 is 70.
2. 67 can be expressed as a difference $70 - 3$
 - (a) Square $70^2 = 4900$
 - (b) Square $3^2 = 9$
 - (c) Find the product of $2 \times 3 \times 70 = 420$
3. $4900 + 9 - 420 = 4489$

Multiplying 2 Digit Numbers Up to 100

Example Evaluate 21×31

1. Multiply the **first digit** of the **first number** by the **first digit** of the **second number** i.e. 2×3 . Place it in front as our first digit.

$$\begin{array}{c}
 \text{21} \times \text{31} \\
 \text{2} \times \text{3} = \text{6} \\
 \text{6 is the first digit} \\
 \text{6} \quad _ \quad _
 \end{array}$$

2. Multiply the **last digit** of the **first number** by the **last digit** of the second number 1×1 . Place it at the back as our last digit

$$\begin{array}{r}
 \text{21} \times \text{31} \\
 \text{1} \times \text{1} = \text{1} \\
 \text{1 is the last digit} \\
 \text{6} \text{ ___ } \text{1}
 \end{array}$$

3. Multiply the inner digits and outer digits of the two numbers and add them $3 \times 1 + 1 \times 2$. Place the sum between the first and last digit.

$$\begin{array}{r}
 \text{21} \times \text{31} \\
 \text{1} \times \text{3} + \text{2} \times \text{1} = \text{5} \\
 \text{6} \text{ 5 } \text{1}
 \end{array}$$

Example. 42×63

1. We multiply the **first digit** of the **first number** by the **first digit** of the **second number** i.e. $4 \times 6 = 24$.

$$\begin{array}{r}
 \text{42} \times \text{63} \\
 \text{4} \times \text{6} = \text{24} \\
 \text{24 is the first two digit} \\
 \text{24} \text{ ___ } \text{___}
 \end{array}$$

2. We multiply the **last digit** of the **first number** by the **last digit** of the **second number** i.e. $2 \times 3 = 6$. Place it at the back as our last digit.

$$\begin{array}{r}
 \text{42} \times \text{63} \\
 \text{2} \times \text{3} = \text{6} \\
 \text{6 is the last digit} \\
 \text{24} \text{ ___ } \text{6}
 \end{array}$$

3. Multiply the **inner digits** and **outer digits** of the two numbers and add them $4 \times 3 + 2 \times 26 = 12 + 12 = 24$. Place the sum between the first and last digit. However, notice that 24 is greater than 10, so we have to carry the 2 over.

$$\begin{array}{r}
 \text{42} \times \text{63} \\
 \text{2} \times \text{6} + \text{4} \times \text{3} \\
 = \text{12} + \text{12} \\
 = \text{24} \\
 \text{24} \text{ 24} \text{ 6}
 \end{array}$$

We can only have a single digit in the middle so we have to carry the 2 over

$$\begin{array}{r}
 \text{24} + \text{4} \text{ 4} \text{ 6} \\
 \text{28} \text{ 4} \text{ 6}
 \end{array}$$

We carry the 2 over by adding it to 24

Exercise. Evaluate the following

1. 21×22

4. 42×24

2. 25×31

5. 65×14

3. 34×13

6. 87×53

Problem Set

1. Multiplying by 5

(a) 6×5

(d) 42×5

(b) 17×5

(e) 99×5

(c) 26×5

(f) 75×5

2. Division by 5

(a) $660 \div 5$

(d) $70 \div 5$

(b) $110 \div 5$

(e) $240 \div 5$

(c) $145 \div 5$

(f) $625 \div 5$

3. Multiplication of 2 digit number with 11

(a) 86×11

(d) 23×11

(b) 15×11

(e) 78×11

(c) 57×11

(f) 55×11

4. Multiplication of any number with 11

(a) 1234×11

(d) 823×11

(b) 5890×11

(e) 881×11

(c) 4583×11

(f) 2401×11

5. Multiplication of 9, 99, 999...

(a) 26×9

(d) 72×99

(b) 89×9

(e) 34×99

(c) 890×9

(f) 93×999

6. Squaring a number ending in 5.

(a) 85^2

(d) 145^2

(b) 55^2

(e) 245^2

(c) 295^2

(f) 95^2

7. Squaring any Number

(a) 20^2

(d) 16^2

(b) 19^2

(e) 53^2

(c) 32^2

(f) 27^2

8. Product of Two Numbers

(a) 83×45

(d) 82×14

(b) 28×31

(e) 35×34

(c) 45×12

(f) 32×85