



Grade 7/8 Math Circles

November 22 & 23, 2016

Math Jeopardy

Let's play Math Jeopardy! Today, we will be playing a fun game of jeopardy to review what you have learned so far.

Round One

Mathematical Puzzles

\$100 Find three positive whole numbers that have the same answer when added together or when multiplied together.

- \$200**
1. A number has 3 digits and is odd
 2. Two digits are the same
 3. The sum of the digits in the tens and ones places is odd
 4. The sum of the digits is 4

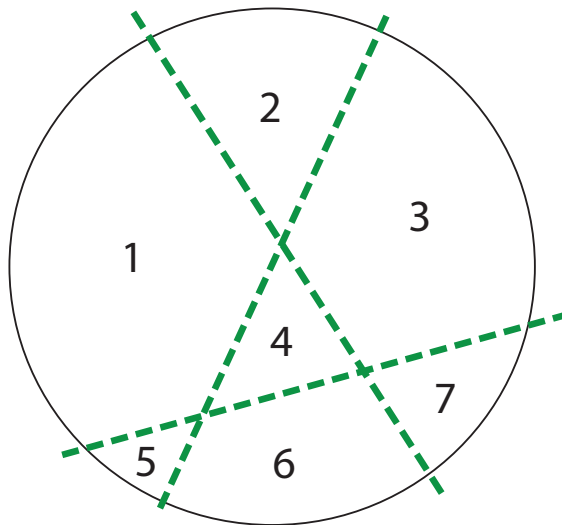
What is the number?

\$300 **Matching Socks**

Sixteen red socks and sixteen blue socks are mixed up in a dresser drawer. The socks are all identical except for their colour. Suppose Richard wants two matching socks but there is a black out so the room is dark and he can't see. What is the smallest number of socks that Richard must take out of the drawer to guarantee he has a pair of socks that match?

\$400 Cutting the Pie!

With one straight cut, you can slice a pie into 2 pieces. With two straight cuts, you can slice a pie into 4 pieces. With three straight cuts, you can slice a pie into as many as 7 pieces, as shown below.



What is the largest number of pieces that you can make with six straight cuts?

\$500 Toothpick Geometry

Make the fish swim in the opposite direction by moving exactly 3 toothpicks.



Continued Fractions

\$100 Express the following as a single improper fraction.

$$\frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}$$

\$200 Find the $\gcd(1428, 546)$ and simplify $\frac{546}{1428}$ to its lowest terms.

\$300 Express $[1; 4, 5, 2]$ as a single improper fraction.

\$400 Express $\frac{159}{43}$ as a continued fraction.

\$500 Solve for x , y , and z .

$$\frac{21}{16} = x + \frac{1}{y + \frac{1}{z}}$$

Sequences

\$100 $\left\{ \frac{8}{3}, \frac{4}{3}, \frac{2}{3}, \frac{1}{3}, \dots \right\}$ is a _____ sequence with a constant _____ of _____.

\$200 Which term in the following sequence is equal to 102?

$$\{4, 11, 18, 25, \dots\}$$

\$300 Find the 5^{th} term of the following recursive sequence.

$$t_n = 2t_{n-1} + 5t_{n-2}; t_1 = 4, t_2 = 2$$

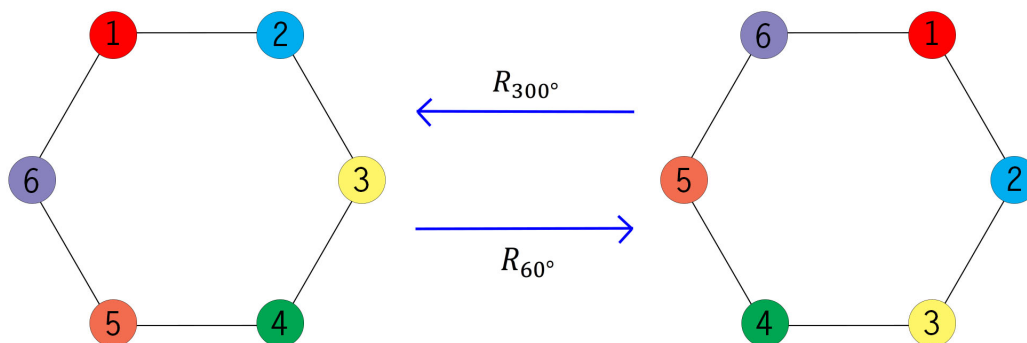
\$400 What is the 3^{rd} term in the arithmetic sequence where $t_{15} = 161$ and $t_{32} = 348$?

\$500 Without using a calculator, what is the last digit in the 56^{th} term in the following sequence?

$$\{9^1, 9^2, 9^3, 9^4, \dots\}$$

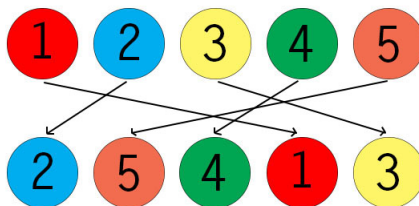
Visual Group Theory

\$100 The hexagon below shows a rotation of 60° from the initial starting position, then a rotation of 300° which undoes the first action.



Draw the hexagon after the following two rotations: $R_{60^\circ} R_{180^\circ}$

\$200 Express this rearrangement mathematically.



\$300 Suppose you have the following five balls.



Draw the final configuration of where the five balls are given the following rearrangement rule:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix}$$

\$400 Determine the equivalent action of

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix}$$

\$500 Create a rearrangement rule to return all the balls to their initial position given the rearrangement rules below.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix}$$

The initial position of the balls is given below.



Random Questions I

\$100 There are three people at the dinner table. Two are mothers, and two are daughters. How is this possible?

\$200 Express the following as a single improper fraction.

$$2 + \frac{1}{5 + \frac{1}{1 + \frac{1}{7}}}$$

\$300 Young Tony Stark asked his grandmother how old she was. Knowing Tony to be quite bright, she replied:

“Tony, I have four children born three years apart between each one and the next. I was 24 when I had my oldest son. Now my youngest is 31. That’s all I’m telling you!”

How old is Tony’s grandmother?

\$400 There are three gentlemen in a meeting: Mr. Red, Mr. Green, and Mr. Gold. They are wearing red, green and gold ties.

Mr. Red: “How amazing! Our last names are Red, Green, and Gold, and one of us is wearing a red tie, another is wearing a green tie, and another is wearing a gold tie.”

Mr. Green: “And none of our tie colours match our names!”

Mr. Gold: “You are right!”

If Mr. Red’s tie is not gold, what is the tie colour of each person?

\$500 Complete the Kenken!

2 /		1 -	
4 ×	2 -		1 -
	2 /		
5 +		4 ×	

Gauss Prep

\$100 Daniel rode his bicycle at a constant speed. After 40 minutes, he cycled 24 km. How far did he cycle in 30 minutes?

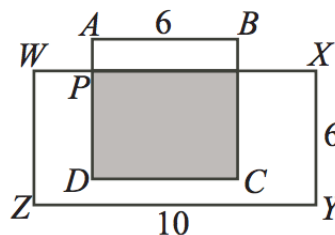
\$200 Joe is reading a 400 page book. On Monday, he reads 40 pages. On each day after the first, the number of pages that he reads is 20 more than on the previous day. Joe finishes the book on _____.

\$300 Which of these values is the largest?

- (a) $\frac{4}{2 - \frac{1}{4}}$ (b) $\frac{4}{2 + \frac{1}{4}}$ (c) $\frac{4}{2 - \frac{1}{3}}$ (d) $\frac{4}{2 + \frac{1}{3}}$ (e) $\frac{4}{2 - \frac{1}{2}}$

\$400 In how many ways can 101 be expressed as the sum of two integers, both greater than zero, with the second integer greater than the first?

\$500 In the diagram, $ABCD$ is a square with side length 6, and $WXYZ$ is a rectangle with $ZY = 10$ and $XY = 6$. Also, AD and WX are perpendicular. If the shaded area is equal to half of the area of $WXYZ$, the length of AP is _____.



Round Two

The Matrix

\$200 Add the following matrices:

$$\begin{bmatrix} 4 & -2 \\ 3 & \frac{1}{2} \\ -8 & 7 \end{bmatrix} + \begin{bmatrix} 9 & 11 \\ 2 & \frac{1}{2} \\ -2 & 8 \end{bmatrix}$$

\$400 Evaluate.

$$2 \begin{bmatrix} 7 & 2 & 21 \\ -4 & 5 & 2 \\ 15 & 3 & -8 \end{bmatrix} - \begin{bmatrix} 7 & 6 & 15 \\ -10 & 10 & 3 \\ 22 & 8 & 4 \end{bmatrix}$$

\$600 Evaluate.

$$\begin{bmatrix} 1 & -3 & 4 \\ \frac{1}{3} & 5 & 7 \end{bmatrix} \begin{bmatrix} -12 & 6 \\ 2 & -5 \\ 1 & 7 \end{bmatrix}$$

\$800 Scott, Hugh, and Peter are doing extra chores to earn some extra allowance this weekend.

- On Friday, they all worked for 2 hours.
- On Saturday, Scott and Hugh worked for 4 hours, and Peter worked for 5 hours.
- On Sunday, Scott worked for 4 hours, and Hugh worked for 5 hours, and Peter worked for 3 hours.

If Scott earns \$5 per hour, Hugh earns \$6 per hour, and Peter works \$7 per hour, on which day did they earn the most money altogether?

\$1000 Hill Cipher

Given the key matrix below, encrypt the following message: **CIRCLE**

$$\begin{bmatrix} 5 & 2 & 6 \\ 1 & 3 & 7 \\ 9 & 5 & 4 \end{bmatrix}$$

Combinatorial Counting

\$200 Teodora is getting a new phone! She is deciding between 4 different smartphones and a red, blue, yellow, or green phone case. How many smartphone and case combinations can she choose from?

\$400 Calculate the following:

$$\frac{12!}{14!}$$

\$600 There's a crisis on Earth! Given a team has 4 members, how many different team arrangements can Batman send from the Justice League if there are 35 available superheroes?

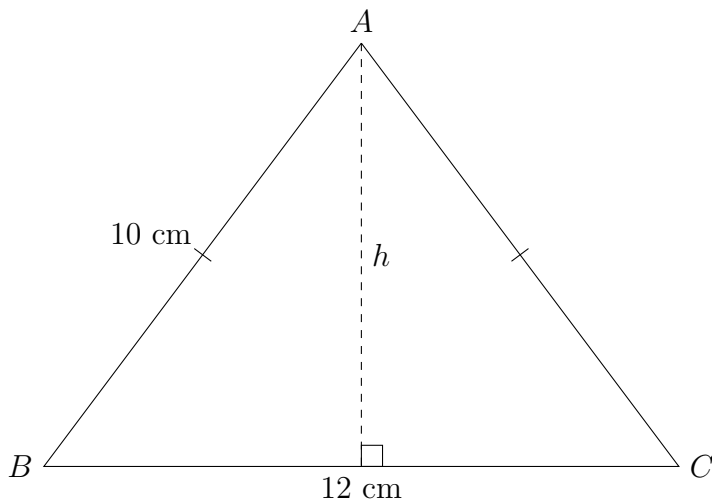
\$800 Calculate the following:

$$\frac{{}_9C_4}{{}_8P_3}$$

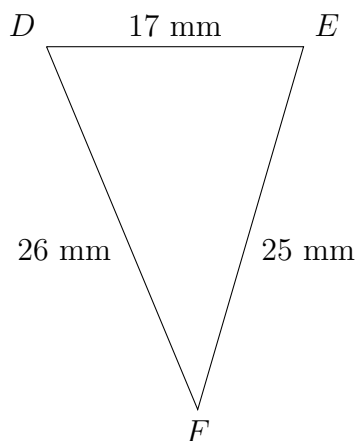
\$1000 Tommy, Chuckie, and three other friends are going to the movies to watch *Fantastic Beasts and Where To Find Them*. But Tommy and Chuckie had a fight and refuse to sit next to each other. How many different ways can Tommy, Chuckie, and friends sit in the movie theatre?

Areas of Triangles

\$200 Find the area of $\triangle ABC$.



\$400 Find the area of $\triangle DEF$.

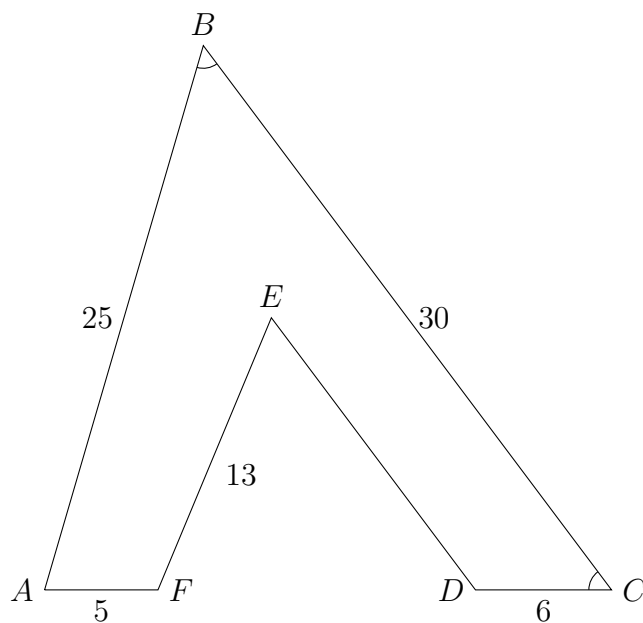


\$600 Given the coordinates below, find the area of $\triangle PQR$.

$$P(3, 13), Q(12, 1), R(28, 17)$$

\$800 The perimeter of $\triangle XYZ$ is 72 cm. If $XY = XZ$ and $YZ = 20$ cm, what is the area of $\triangle XYZ$?

\$1000 Given that the perimeter of $\triangle DEF$ is 42, what is the area of hexagon $ABCDEF$?



Logic

\$200 James made some cookies. He ate one cookie and gave half of the rest to Guntaas. Then he ate another cookie and gave half of the rest to Daniel. James now has 5 cookies. How many cookies did James start with?

\$400 There are three Dalmatian puppies: Spot, Socks, and Patches. Spot has fewer spots than Socks, but more spots than Patches. Which puppy is the spottiest?

\$600 How can you use only five 5s and only addition to make 565?

\$800 Fill in the blanks!

$$\begin{array}{r} \square \square \square \\ \times \quad \square \square 2 \\ \hline \square \square 5 \square \\ + \square \square \square 0 \\ \hline \square 3 \square 9 \square 1 \square 2 \end{array}$$

\$1000 Let's play Clue! Miss Scarlet, Colonel Mustard, Mr. Green, Mrs. Peacock and Mrs. White were involved in a theft. One of the five stole a credit card from one of the other four. The following facts are known:

1. A man and woman were eating at McDonalds at the time of the theft
2. The thief and victim were together at the bank at the time of the theft
3. Colonel Mustard was not with a married woman at the time of the theft
4. Mr. Green was not with Mrs. White at the time of the theft
5. One of the married women was alone at the time of the theft
6. One of the married women was at the bank at the time of the theft
7. The victim is a man

Who is the thief and who is the victim?

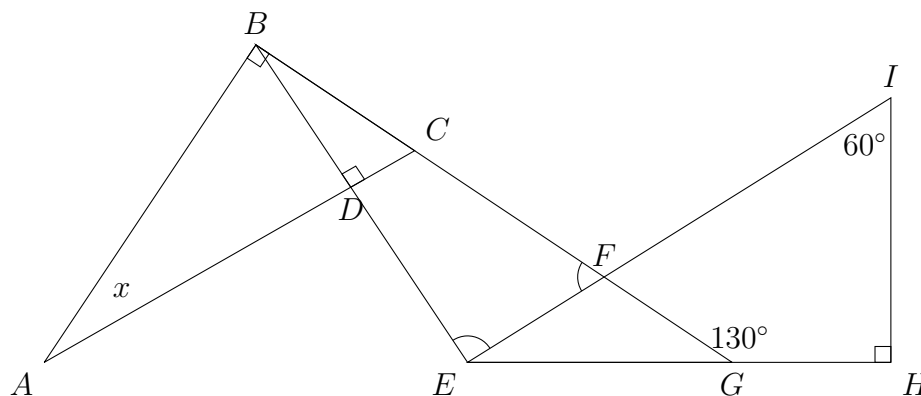
Random Questions II

\$200 What are the missing values for x , y , and z ?

$$\begin{bmatrix} 15 & 4 \\ 23 & x \end{bmatrix} - 3 \begin{bmatrix} y & 3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ z & 11 \end{bmatrix}$$

\$400 Which one is the better deal? 10 chocolates for \$15 or 5 chocolates for \$8?

\$600 What is the measure of angle x ?



\$800 If the area of a square is 64 m^2 , what is the perimeter?

\$1000 There are 52 cards in a standard deck of cards. How many ways can Jiin select 4 cards from the deck if 2 of the cards are spades? (**Hint:** There are 13 cards per suit.)

Gauss Prep

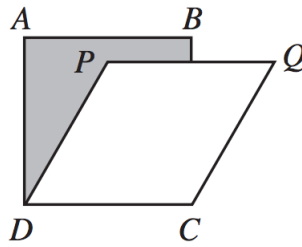
\$200 If $10^x - 10 = 9990$, then x is equal to _____

\$400 In a class of 40 students, 18 said they liked apple pie, 15 said they liked chocolate cake and 12 said they did not like either. How many students in the class liked both?

\$600 Winifred earns \$10/hour and works 8 hours per day for 10 days. She first spends 25% of her pay on food and clothing, and then pays \$350 in rent. How much of her pay does she have left?

\$800 The values of r , s , t , and u are 2, 3, 4, and 5, but not necessarily in that order. What is the largest possible value of $r \times s + u \times r + t \times r$?

\$1000 In the diagram, $ABCD$ is a square with area 25 cm^2 . If $PQCD$ is a rhombus with area 20 cm^2 , the area of the shaded region, in cm^2 , is _____



Final Jeopardy

A rectangular piece of paper $ABCD$ is folded so the edge CD lies along edge AD , making a crease DP . It is unfolded, and then folded again so that edge AB lies along the edge AD , making a second crease AQ . The two creases meet at R , forming triangles PQR and ADR , as shown. If $AB = 5$ cm and $AD = 8$ cm, what is the area of quadrilateral $DRQC$, in cm^2 ?

