Intermediate Math Circles
Wednesday, February 8, 2017
Problem Set 1

1. (a) Draw a graph whose vertices and edges are given by

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

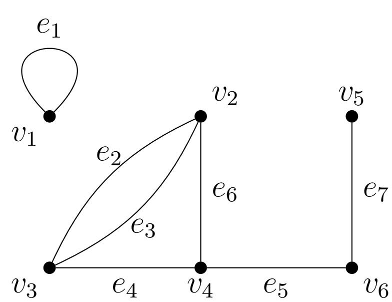
$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

$$\begin{aligned} e_1 &= \{v_1, v_2\} \\ e_2 &= \{v_1, v_3\} \\ e_3 &= \{v_2, v_3\} \\ e_4 &= \{v_2, v_2\} \\ e_5 &= \{v_3, v_4\} \\ e_6 &= \{v_5, v_6\} \end{aligned}$$

(b) Draw a simple graph G with 4 vertices and 6 edges. Find a way to draw this graph so that no two edges cross.

(c) Draw a simple, connected graph with 6 vertices and 7 edges such that the removal of one of the edges disconnects the graph.

2. In the following graph G, find an example of a path of length 4, and an example of a walk of length 3 that is *not* a path. List the degree for each vertex in G. Is G connected?



3. Determine the number of edges in each of the following graphs:

(a) N_n

(b) C_n with $n \geq 3$

(c) W_n with $n \geq 4$

(d) S_n

(e) K_n

(f) $K_{m,n}$ with $m, n \geq 1$

(g) A k-regular graph on n vertices.

4. For each graph N_n, C_n, W_n, and S_n, determine the values of n for which the graph is bipartite.

5. Given a positive integer k, explain how one would construct a 3-regular graph on $2k$ vertices. Draw such a graph when $k = 4$.

6. Why must every graph have an even number of vertices of odd degree?

7. Construct a simple graph on $n \geq 2$ vertices such that no two vertices have the same degree or argue that such a graph cannot exist. What if the graph is not simple?

8. Let G be a simple graph on $n \geq 2$ vertices. Suppose that for every pair of distinct vertices u, v in G, we have

$$\deg(u) + \deg(v) \geq n - 1.$$

Show that G must be connected.