# Intermediate Math Circles
# Wednesday, March 1, 2017
# Sequences and Series I

## What Comes Next?

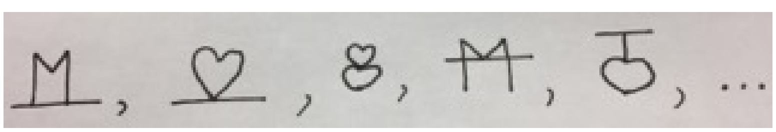
Find a pattern and give the next three numbers/objects in the sequence according to your pattern. (There could be more than one answer.)

1. 4, 7, 10, 13, 16, . . .

2. 2, 10, 50, 250, 1250, . . .

3. 1, 4, 10, 22, 46, 94, . . .

4. 1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211, . . .

5. 

## Sequence:

A **sequence** is an ______________________________. Our objects will usually be numbers. The numbers in the list are called the **terms** of the sequence.
We are especially interested in sequences where the terms of the sequence can be determined using some sort of rule or pattern.
We often use subscripts to indicate the term's place in the sequence. We label the $n^{\text{th}}$ term in the sequence as _____.

For example, if our sequence is $4, 7, 10, 13, 16, 19, 22, 25, \ldots$, then $t_6 =$ _____.

## Arithmetic Sequence:

An **arithmetic sequence** is a sequence with the property that ______________________________.

This difference is called the ______________________.

**Example:** $4, 7, 10, 13, 16, \ldots$ is an arithmetic sequence with a common difference of _____.

What is the general term, $t_n$, of an arithmetic sequence?

Let $a$ be the first term of the sequence and let $d$ be the common difference. Then

$t_n =$ _________________________________.

**Practice:**

1. If the first three terms of an arithmetic sequence are $49, 41, 33$, then what is the tenth term?

2. In an arithmetic sequence, the third term is $75$ and the eleventh term is $131$. Find the eighth term.

## Geometric Sequence:

A **geometric sequence** is a sequence with the property that ______________________________.

This ratio is called the ______________________.

**Example:** $2, 10, 50, 250, 1250, \ldots$ is a geometric sequence with a common ratio of _____.

What is the general term, $t_n$, of a geometric sequence?

Let $a$ be the first term of the sequence and let $r$ be the common ratio. Then

$t_n =$ _________________________________.

**Practice:**

1. If the first three terms of an geometric sequence are $\frac{3}{2}, \frac{1}{2}, \frac{1}{6}$, then what is the eighth term?

2. The fifth term of a geometric sequence is 48 and its common ratio is 2. Find the general term.

## Sequences that Repeat:

We can form a sequence by repeating a finite sequence.
**Example:** $2, 4, 6, 8, 2, 4, 6, 8, 2, 4, 6, 8, \ldots$ is a sequence that is formed by repeating the finite

sequence ______________________________

**Practice:**

1. A sequence is formed by endlessly repeating the finite sequence $1, 3, 5, 7, 9$. What is the $2017^{\text{th}}$ term of the sequence?

2. Find the units digit of $3^{35}$.

## Recursive Sequences:

A **recursive sequence** is a sequence in which the terms (except for the initial term(s)) are defined using ________________________.

**Example:** We define a recursive sequence as $t_1 = 1$, $t_2 = 1$ and $t_n = t_{n-1} + t_{n-2}$ for $n \geq 3$.

Using this definition we obtain the sequence ________________________.

This sequence is one of the most famous sequences. It is called the ____________________.

## Arithmetic and Geometric Sequences Defined Recursively:

How do we define an arithmetic sequence recursively? ________________________.

How do we define a geometric sequence recursively? ________________________.

**Practice:**

1. Given $t_1 = 1, t_2 = 2, t_3 = 3$ and $t_n = t_{n-1} - t_{n-2} + t_{n-3}$, for $n \geq 4$, find $t_7$.

2. Given $t_1 = 3$ and $t_n = n \cdot t_{n-1}$, for $n \geq 2$, find $t_9$.

3. Find a recursive definition for the sequence $1, 3, -2, 5, -7, 12, -19, 31, \ldots$.

## Online Encyclopedia of Integer Sequences (www.oeis.org):

- Look up a sequence given part of the sequence:

  What sequence is this? $61, 52, 63, 94, 46, \ldots$

  ________________________________________.

- Find the next few terms in a sequence:

  You are counting the number of dots in a triangle. You know the first few are 1, 3, 6, 10, 15 and you want to find the 100th number in the list.

  The answer is ________________________.

- Look up a sequence by name:

  Mersenne primes are primes which are of the form $2^p - 1$.

  $3, 7, 31, 127, 8191, 131071, \ldots$