

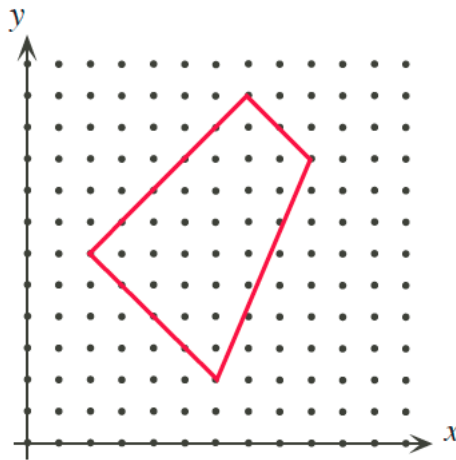


Intermediate Math Circles

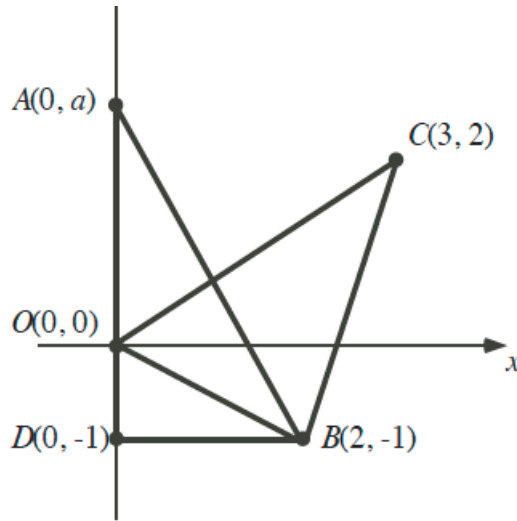
Wednesday, March 22, 2017

Problem Set 6

- Find the coordinates of the point that divides the distance
 - from $(7, 1)$ to $(3, 5)$ internally in the ratio 1:1 (i.e. find the midpoint)
 - from $(-4, 8)$ to $(2, -4)$ internally in the ratio 1:5
 - from $(-4, 8)$ to $(2, -4)$ internally in the ratio 2:1
 - from $(4, -5)$ to $(-1, 2)$ internally in the ratio 3:2
- Find the coordinates of point P which divides the length from $A(4, -2)$ to $B(-6, 8)$ externally in the ratio of 3 : 1. By externally, I mean find point P on the same line as AB, but beyond the line segment AB.
 - For the same points A & B above, find the coordinates of point Q which divides the length of AB externally in the ratio of 1 : 3
- Show that the line joining the midpoints of the two sides of triangle with vertex $(1, 1)$ is parallel and equal to half the base $(-5, -1), (3, -9)$.
- Given the points $A(-3, 0), B(-2, -5), C(2, 1)$, find a point P that $PA \parallel BC$ and $PC \perp BC$.
- 2015-16 Problem of the Week (POTW)- Level C- Week 10
The dots on the diagram are one unit apart, horizontally and vertically. Determine the area of the figure.



- 2015-16 POTW- Level D- Week 10
In the diagram, $A(0, a)$ lies on the y-axis above the origin. If $\triangle ABD$ and $\triangle COB$ have the same area, determine the value of a .



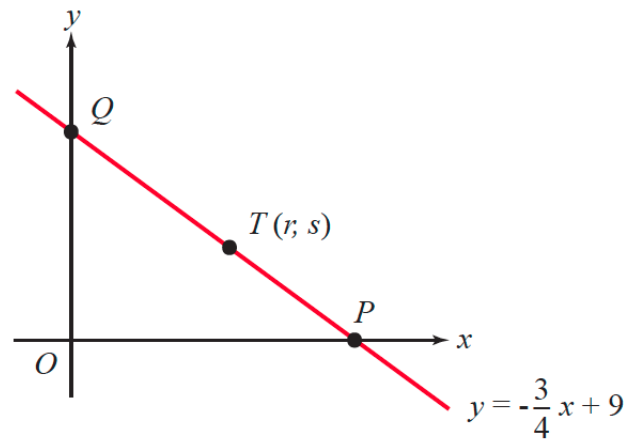
7. 2013-14 POTW- Level D

The line $y = -\frac{3}{4}x + 9$ crosses the x-axis at P and the y-axis at Q.

Point $T(r, s)$ lies on the line segment PQ.

The area of $\triangle POQ$ is three times the area of $\triangle TOP$.

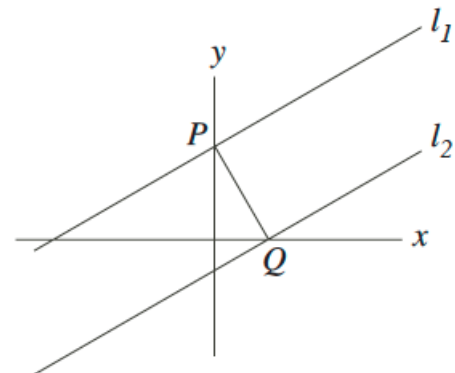
Determine the values of r and s , the coordinates of T.



8. 2010-11 POTW- Level D

Line l_1 has equation $y = mx + k$. Line l_1 crosses the y-axis at point P and l_2 crosses the x-axis at the point Q. PQ is perpendicular to both line l_1 and line l_2 .

Determine the y-intercept of l_2 in terms of m and k .



9. 2012 Canadian Team Mathematics Contest Individual Problems Q3

The line $x = 2$ intersects the lines $y = -2x + 4$ and $y = \frac{1}{2}x + b$ at points a distance of 1 unit from each other. What are the possible values of b ?

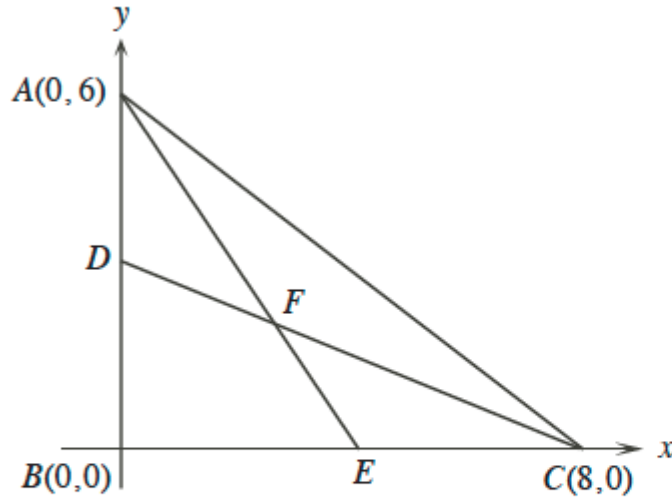
10. 2013 Canadian Team Mathematics Contest Individual Problems Q3

Find the area of the triangle formed by the lines $4x - 3y = 0$, $x + 2y = 11$ and the x-axis.



11. 2011 Hypatia Q1

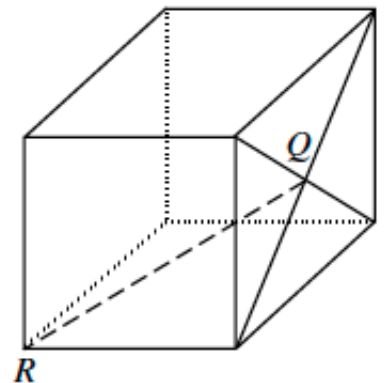
In the diagram, D and E are the midpoints of AB and BC respectively.



- (a) Determine an equation of the line passing through the points C and D .
- (b) Determine the coordinates of F , the point of intersection of AE and CD .
- (c) Determine the area of $\triangle DBC$.
- (d) Determine the area of quadrilateral $DBEF$.
12. Find the length of the following line segments between points $O(0, 0, 0)$, $A(-6, 1, 0)$, and $B(0, 3, -4)$.
- (a) OA
- (b) BA
13. A square-based pyramid has a height of $\sqrt{31}$ m and a base area of 100m^2 . What is the length of the slant of the pyramid (length from the corner of the base to the top of the pyramid)?
14. 1998 Pascal Q21

Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of QR is

- (A) 2 (B) $\sqrt{8}$ (C) $\sqrt{5}$
(D) $\sqrt{12}$ (E) $\sqrt{6}$

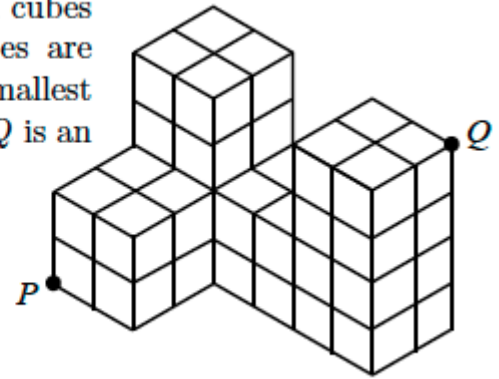


15. 2014 Pascal Q23



In the diagram, the shape consists of 48 identical cubes with edge length \sqrt{n} . Entire faces of the cubes are attached to one another, as shown. What is the smallest positive integer n so that the distance from P to Q is an integer?

- (A) 17
- (B) 68
- (C) 7
- (D) 28
- (E) 3



16. 2013 Pascal Q22

A water tower in the shape of a cylinder has radius 10 m and height 30 m. A spiral staircase, with constant slope, circles once around the outside of the water tower. A vertical ladder of height 5 m then extends to the top of the tower. Which of the following is closest to the total distance along the staircase and up the ladder to the top of the tower?

- (A) 72.6 m
- (B) 320.2 m
- (C) 74.6 m
- (D) 67.6 m
- (E) 45.1 m

