

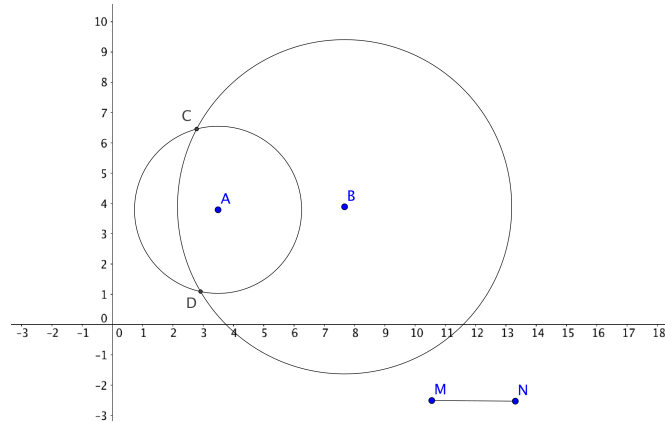


Intermediate Math Circles

Wednesday, March 29, 2017

Problem Set 7

1. Using GeoGebra(*geogebra.org*), determine the locus of points that are twice as far from point A as they are from point B.

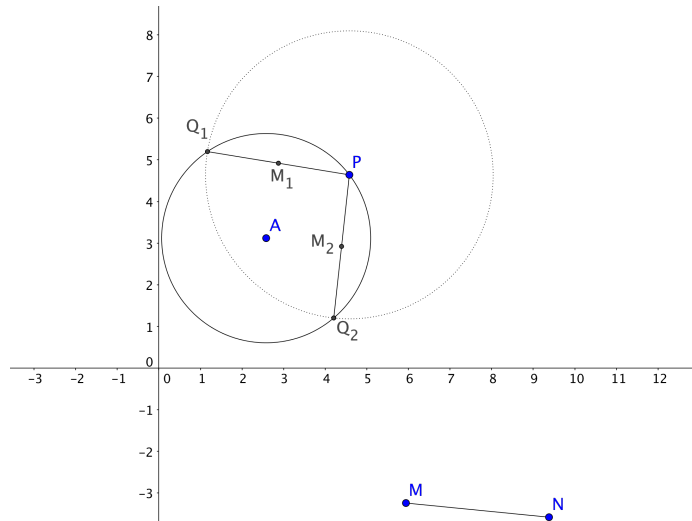


Steps:

- Construct and label two points A and B.
- Construct a line segment of arbitrary length. Label the end points M and N.
- Construct a circle with centre A and radius MN .
Note: Can do this using the *Input:* bar and the command **Circle**[<Point>,<Radius Number>].
- Construct a circle with centre B and radius twice the length of MN.
- Select the points of intersection of the two circles and label them C and D.
Note: You may need to adjust the length of line segment MN so that the circles intersect.
- Right click on points C and D and select *Trace On*.
- Vary the length of line segment MN.

Questions:

- Describe the locus
 - Change the location of point A. Describe how the locus changes
 - when points A and B are closer together
 - when points A and B are farther apart
2. Using GeoGebra(*geogebra.org*), consider chords of equal length drawn in a circle. Determine the locus of the midpoints of the chords.



Steps:

- i Construct a line segment MN . This will be the length of the chord.
- ii Construct a circle with centre A and point P .
Hint: the command **Circle**[<Point>,<Point>] will be helpful
- iii Construct a circle with centre P and radius of length MN .
Hint: remember command **Circle**[<Point>,<Radius Number>]
- iv Call the intersections of your two circles Q_1 and Q_2 .
Note: You can hide your recently created circle by right clicking on the circle and unselecting *Show Object* and *Show Label*.
- v Using the line segment command create cords PQ_1 and PQ_2 .
- vi Construct the midpoints of line segments PQ_1 and PQ_2 . Rename the midpoints M_1 and M_2 .
- vii Right click on points M_1 and M_2 and select *Trace On*.
- viii Vary the length of line segment MN .

Questions:

- (a) Describe the locus of midpoints of the chords
 - (b) Where is do you suspect the centre of the locus is located?
 - (c) How would the locus change if you only had one of M_1 and M_2 ?
3. Given the points $A(2, 0)$ and $B(5, 0)$, find the equation of the locus of points that are twice as far from point A as they are from point B .



4. Determine an equation for each for the following circles
- (a) centre $(0, 0)$, through $(-2, 3)$
 - (b) centre $(0, 0)$, x-intercepts at ± 8
 - (c) centre $(3, 4)$, through $(0, 0)$
 - (d) centre $(-1, 3)$, through $(1, -1)$
 - (e) centre $(-2, -2)$, y-intercept -2
5. (a) Show that the points $P(-2, 4)$ and $Q(2, -4)$ are both on the circle $x^2 + y^2 = 20$.
(b) Show that PQ is a diameter of the circle
6. Determine the equations of the circles with the given diameters
- (a) from $(-3, 5)$ to $(3, -5)$
 - (b) from $(-1, 2)$ to $(5, 8)$
7. For the circle given by $x^2 + y^2 = 34$,
- (a) show that the line segment from $P(-5, 3)$ to $Q(3, 5)$ is a chord of the circle;
 - (b) find the midpoint M of the chord;
 - (c) show that $MO \perp PQ$
8. A circle passes through the points $A(-1, 1)$ and $B(6, 0)$ and has its centre on the line $x + 3y + 7 = 0$. Find the equation of the circle.