Intermediate Math Circles
Wednesday, March 8, 2017
Sequences and Series II

Series:

A **series** is a sum of the terms of a sequence.

Consider the sequence $1, 2, 3, 4, 5, 6, \ldots$. What is the sum of the first 100 terms of this sequence?

What is the sum of the first n terms of this sequence?

What is the sum of all the terms from the 50th term until the 150th term?

Summation Notation:

The notation

$$\sum_{i=m}^{n} t_i$$

is called **summation notation** and it represents the sum

$$t_m + t_{m+1} + t_{m+2} + \cdots + t_n$$

The summation symbol, $\sum$, is the upper case Greek letter __________.

The letter i is the ____________________.

The letter m is the ____________________.

The letter n is the ____________________.

The expression $i = m$ under the summation symbol means that the index i begins with an initial value of m and increments by 1 and stops when $i = n$.

The index of summation is a *dummy* variable and any letter could be used in its place.

Evaluating Sums in Summation Notation:

Example: Evaluate the following sums.

1. $\displaystyle\sum_{i=1}^{5} i^2 =$

2. $\displaystyle\sum_{k=4}^{6} (k^2 + k) =$

Arithmetic Series:

Recall: An **arithmetic sequence** is ____.

Let a be the first term of the sequence and let d be the common difference. Then the nth term in the sequence is ________________________.

The sum of the first n terms of an arithmetic sequence is

$$\sum_{i=1}^{n} t_i =$$

Practice:

1. Determine the sum of the first 50 positive even integers.

2. Evaluate $\sum_{i=15}^{80} 5i$.

Geometric Series:

Recall: A **geometric sequence** is ____.

Let a be the first term of the sequence and let r be the common ratio. Then the nth term in the sequence is ________________________.

The sum of the first n terms of a geometric sequence is

$$\sum_{i=1}^{n} t_i =$$

If $r = 1$, then the sum is _______________.

Practice:

1. Evaluate $\displaystyle\sum_{k=1}^{20} \frac{1}{2^{k-1}}$.

2. You are hired to work every day of June. Your boss offers two options:

   (i) You are paid one cent on the first day and every day after that your daily pay doubles.

   (ii) You are paid \$10000 on the first day and every day after that your daily pay goes up by \$1000.

   Which option should you take? (June has 30 days.)

Telescoping Series: A **telescoping series** is a series where ______________________.

Example: Evaluate

$$\sum_{i=1}^{100} \left(\frac{1}{i} - \frac{1}{i+1} \right).$$

Practice

Evaluate the following sums:

1. $\displaystyle\sum_{i=1}^{25} \left(\frac{1}{2^i} - \frac{1}{2^{i+1}} \right)$

2. $\displaystyle\sum_{i=1}^{50} \left[(i+1)^2 - i^2 \right]$

Looking for Patterns:

Evaluate the following sums:

1. $\displaystyle\sum_{i=1}^{100} \frac{1}{i(i+1)}$

2. $\sum_{i=1}^{100} i^3$

How Do We Know Our Formula is True?

We use a way of proving called ______________________________.