UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

# Grade 6 Math Circles

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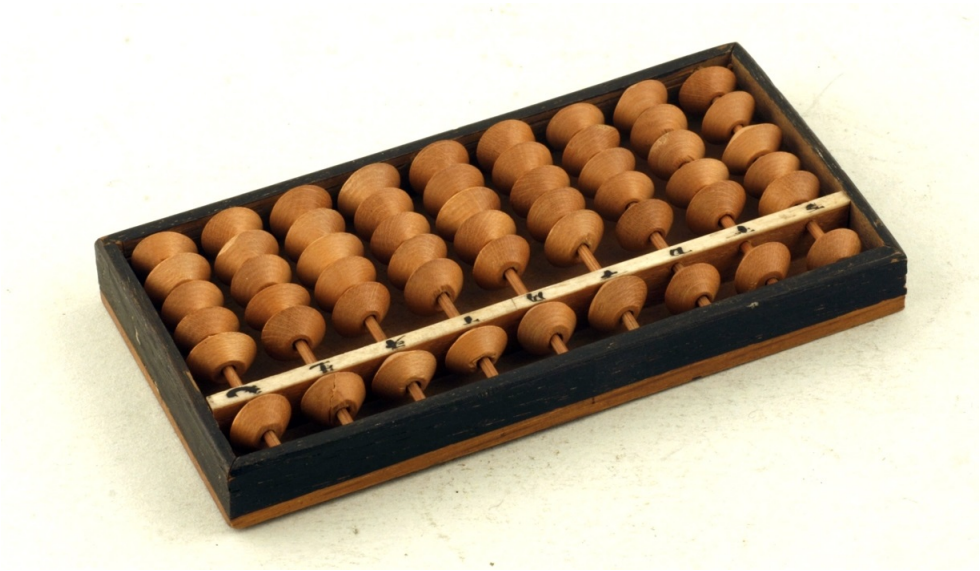
***The Abacus and Counting Rods***

## History of the Abacus

The abacus is known as the oldest calculator but is still often used today!

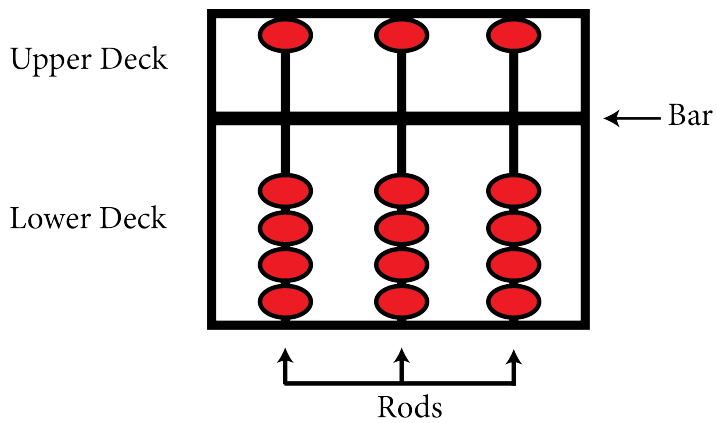
As early as 2700 BCE, the first abacus like counting tables were used in Ancient Mesopotamia, Egypt and Persia before gaining wider popularity in Ancient Greece. Different variations of of the abacus have also appeared in Roman, Indian, Russian, and Native American cultures. The type that most people today are familiar with are the Japanese and Chinese abacuses, called "soroban" and "suanpan" respectively.

Today, we are going to be learning to use the Japanese abacus.



# Make an Abacus

1. Take a popsicle stick frame, 3 pipe cleaners, and a bag of beads.
2. Twist the 3 pipe cleaners around the bottom of the popsicle stick frame. (Hold the frame so the larger area is at the bottom).
3. Put 4 beads on each pipe cleaner and slide them all the way down to the frame.
4. Wrap the pipe cleaners around the middle popsicle stick.
5. Put 1 bead on each pipe cleaner and slide them down to the popsicle stick.
6. Wrap the remainder of the pipe cleaners around the top popsicle stick.

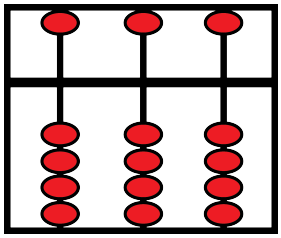



We will call the pipecleaners the **rods** of the abacus and the middle popsicle stick the **bar**. The bottom portion of the abacus (with 4 beads on each rod) is called the **lower deck** and the portion with only 1 bead on each rod is called the **upper deck**.

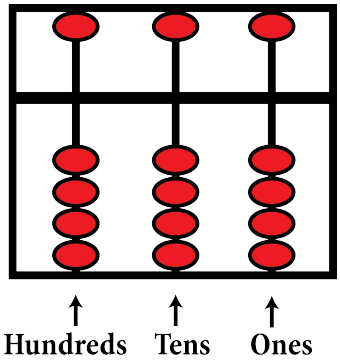
# Setting a Number

Before we start using the abacus to do math, we have to learn how to input numbers!

The **resting position** of the abacus, representing the number 0, is when no beads are touching the bar. This means the beads in the lower deck and slid all the way down and the beads in the upper deck are slid all the way up.

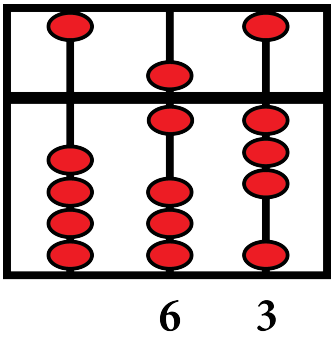


The decimal system that we use to write down numbers is actually very similar to the ways numbers are shown on the abacus! **Each rod represents a place value.** The rod on the right represents the ones digit, the rod in the middle represents the tens digit, and the rod on the left represents the hundreds digit. By adding rods, you can add digits to your calculations.




Each bead on the **lower deck represents 1** of that rod's place value when you slide it up towards the bar. Each bead on the **upper deck represents 5** of the rod's place value when you slide it down towards the bar. When we set the abacus, we always start from the left bar and then go right and when a bead is slid towards the bar, it is put in the **active position**.

**Let's try this out:** $63 = (6 \times 10) + (3 \times 1) = ((5 + 1) \times 10) + (3 \times 1)$

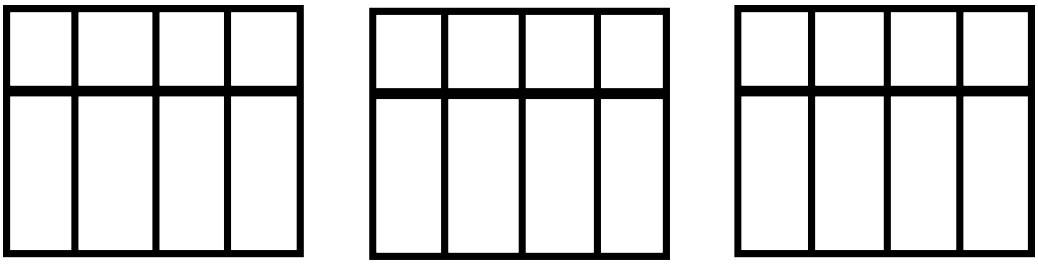



We slide the bead in the upper deck of the middle rod down the the bar to represent 5 tens, and one bead in the lower deck of the middle rod up to the bar to represent 1 tens. On the left rod, we slide 3 beads on the lower deck up to the bar to represent 3 ones.

**Fun Fact:** Traditionally, you should only move beads up with your thumb and down with your index finger.

**Exercise:** Try setting the abacus to these numbers.

8 34 199



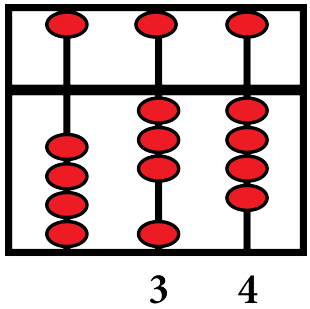
## Addition

Addition on the abacus is fairly straightforward.

1. Start by setting the abacus to the first number.
2. Then, set the abacus to the next number (from left to right).
3. If you need to activate a bead in the upper deck but it is already in active position, then slide it back up to resting position and on the rod to the left (one place value higher), add 1.
4. If you need to slide a bead up in the lower deck but all the beads of the lower deck are already in active position, put all the lower deck beads on that rod back to resting position and the upper deck bead to active position instead of activating one. Then, continue activating beads.
5. The number you now have displayed on your abacus is your sum!

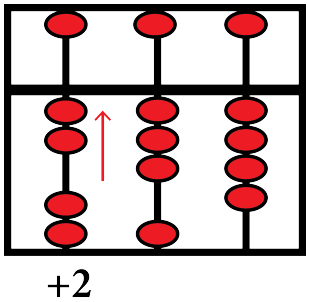
**Let's try it out:** $34 + 217$

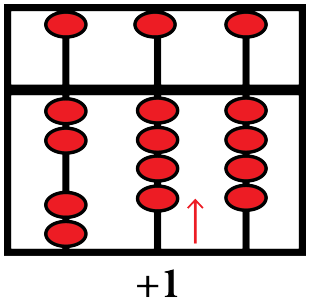
First, set the abacus to 34 from resting position.



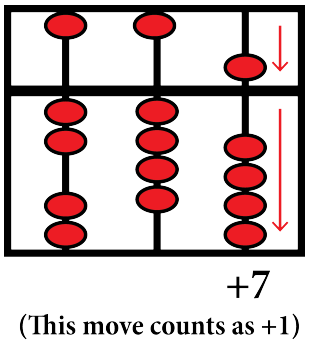

Then, set to 217 starting from the rod on the left.

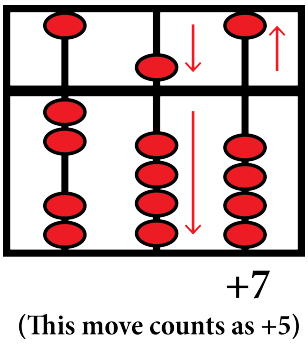
Activate 2 in the hundreds rod. Activate 1 in the tens rod.

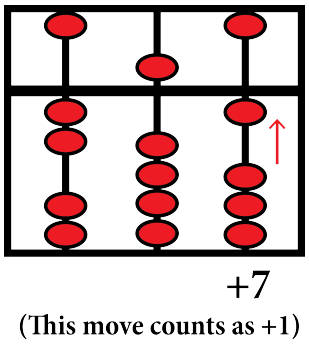



Activate 7 in the ones rod. Remember that $7 = 5 + 2$. This time we have to carry over to the upper deck and to the next rod.




What number does the abacus show now?

**Exercise:** Use the abacus to perform these additions. Only write down the final answer.

1. $75 + 54 =$
2. $249 + 603 =$

## Subtraction

With subtraction, use the terminology **base, subtractant** and **difference**.

$$\text{Base} - \text{Subtractant} = \text{Difference}$$

To do subtraction on the abacus, you must first know the **complementary numbers** that sum to 5 and 10.

Complementary numbers that sum to 5:

- 1 and 4
- 2 and 3

Complementary numbers that sum to 10:

- 1 and 9
- 2 and 8
- 3 and 7
- 4 and 6
- 5 and 5

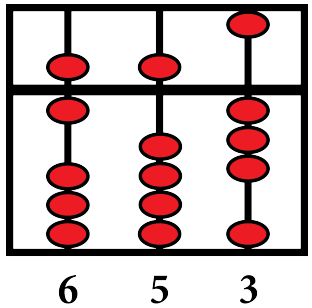
Here is how you subtract on the abacus and how complementary numbers are helpful:

1. Set the abacus to the base from resting position.
2. From left to right, slide beads back into resting position for each rod instead of activating them as you set the subtractant. Make sure there are enough beads activated on that rod before you start sliding them to rest position.
3. If there are not enough beads on a rod's lower deck currently active to subtract, then subtract one bead from the same rod's upper deck and add the **complementary number that sums to 5** to the rod's lower deck.
4. If there are also not enough beads on a rod's upper deck currently active to subtract, then subtract a bead from the lower deck of the rod to the left, and add the **complementary number that sums to 10** to the original rod.

This may seem confusing at first but once you get used to this, you may find it easier than having to remember what remains when you subtract numbers when a base's digit is smaller than the subtractant's!

**Let's try it out:** $653 - 519$

First, set the abacus to 653.

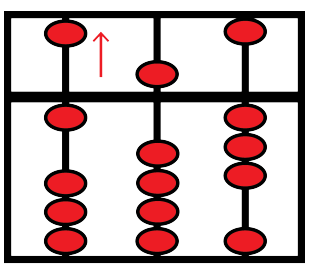



Then, subtract 519 starting from the rod on the left.

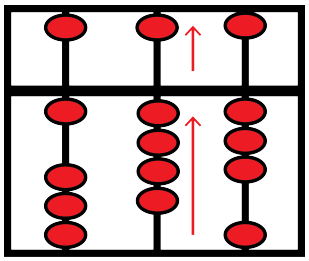
Subtract 5 on the hundreds rod.

Subtract 1 on the tens rod. But there are not enough active beads in the lower deck to put 1 to rest so slide the bead in the upper deck of that rod to rest and add the complimentary number that sums to 5 (which is 4) beads in the lower deck.

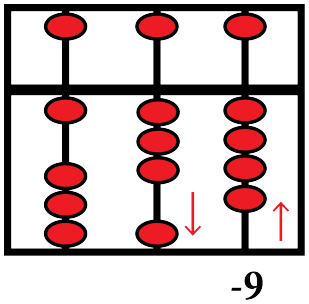
Subtract 9 on the ones rod. Because there are not enough beads in the lower and upper decks of this rod, subtract 1 bead from the rod to the left (the middle rod) and add the complementary number that sums to 10 (which is 1).




(Put a 5 bead back to rest)




(Subtract 5, add 4)




(Subtract 1 on the next rod, add 1)

What number is on your abacus now?

**Exercise:** Use the abacus to perform these subtractions:

1. $16 - 9 =$
2. $246 - 191 =$

# Multiplication

With multiplication, use the terminology **base, multiplier** and **product**.

$$\text{Base} \times \text{Multiplier} = \text{Product}$$

To multiply on the abacus, you first have to know your multiplication table from 0 to 9. Multiplication is more complicated than addition or subtraction so we will focus on multiplying a base by a single digit multiplier. Multiplication with larger numbers is very similar but you would need an abacus with more rods.

1. Start by counting the total number of digits in the two numbers your are multiplying. Then, starting from the ones rod and going left, count that number of rods. This rod is your "current" place holder rod. It is very important to keep track of which rod you are currently on with multiplication.

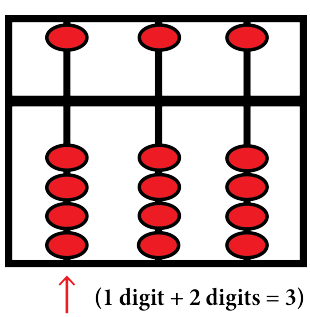
2. For each digit of the base (from left and going right), multiply the digit by the multiplier. Set this product on your abacus while treating your "current" rod as the tens rod and the rod to the left of your "current" rod as the ones rod.

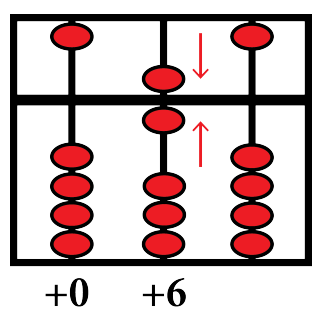
3. Move your "current rod" one rod to the right.

4. Move on to the next digit of your base and repeat these steps until every digit in your base has been multiplied by the multiplier.

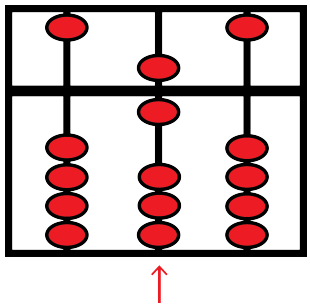
**Let's try it out:** $15 \times 6$

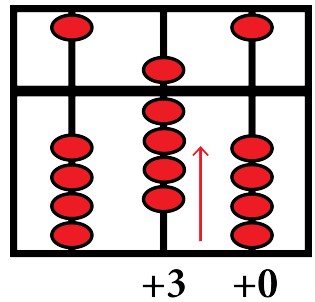
First, count the number of digits in the base and the multiplier and find your current rod. Multiply $1 \times 6 = 6$ so set 06 on the current rod.



Move the current rod to the right by one. Multiply $5 \times 6 = 30$ so set 30 on the current rod.






What is displayed on your abacus now?

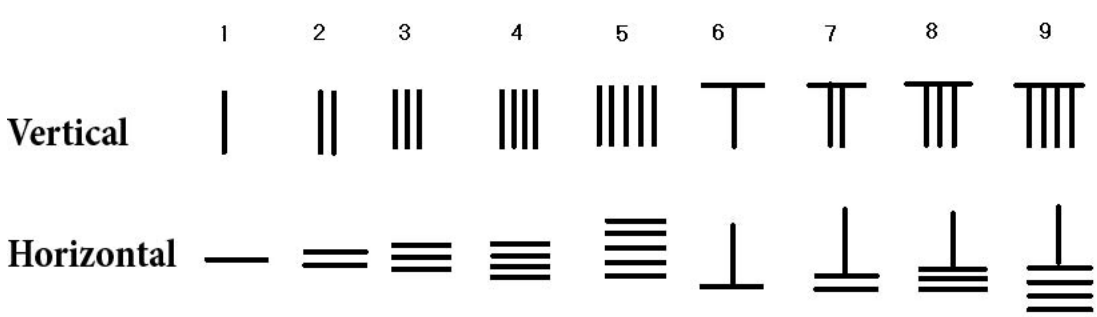
**Exercise:** Use the abacus to perform these multiplications:

1. $7 \times 4 =$
2. $54 \times 8 =$

## Counting rods

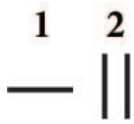
Counting rods (also known as rod numerals) are a system of doing calculations in Ancient China around 475 BCE before the abacus became popular in China. It is a **positional number system** which means it has symbols which represent numbers from 1 to 9 and the symbol's position in the number determines its values (much like the number system we use today). In Ancient China, small wooden rods were used to write these symbols and they were moved around on a counting board to do math.

The numbers 1 to 9 can be represented vertically or horizontally.



The ones place value always uses vertical symbols. When you move up a place value (ie. from ones to tens), you switch between writing the symbols vertically and horizontally. This is done so that the number is easier to read. The counting rod system does not have a symbol for 0 and instead just leaves a blank space where the 0 would be.

So using counting rods, the number 12 would look like:




**Let's try it out!** How do you write the number 2 478 935 using counting rods?

**Exercise:** Write these numbers as counting rods:

1. 8

2. 339

3. 521 674

## Problem Set

1. Set the abacus to these numbers:

   (a) 9

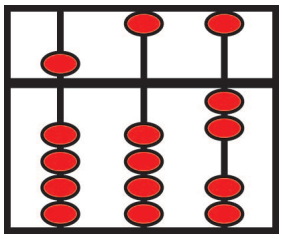
   (b) 31

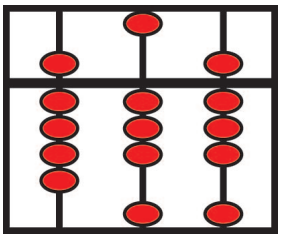
   (c) 847

   (d) 999

   (e) 

2. What number is on each of these abacuses?

   (a) 

   (b) 

3. Find the sums using only an abacus:

   (a) $111 + 6 =$

   (b) $12 + 14 =$

   (c) $904 + 95 =$

   (d) $399 + 47 =$

   (e) 

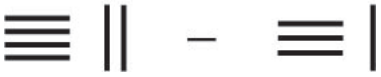
4. Find the differences using only an abacus:

   (a) $349 - 5 =$

   (b) $35 - 34 =$

   (c) $875 - 236 =$

   (d) $201 - 153 =$

   (e) 

5. Find the product using only an abacus:

   (a) $4 \times 5 =$

   (b) $35 \times 8 =$

   (c) $97 \times 4 =$

   (d) $101 \times 7 =$

   (e) 

6. Solve this with the abacus without writing any steps in between. Write only the final answer.

$$11 \times 9 + 302 - 96 - 43 + 16 =$$

7. **Challenge problem**. When we use the abacus, we always go from left to right when we move from rod to rod. What if we went right to left instead? Try the following problems usng both left to right and right to left methods. Does right to left still give the correct answer and why?

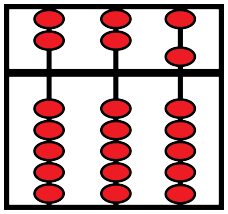
   (a) $35 + 782$

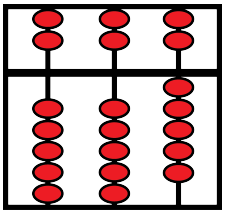
   (b) $909 - 543$

   (c) $96 \times 7$

8. **Challenge problem**. The Chinese abacus is very similar to the Japanese abacus except that it has 2 beads in the upper deck and 5 beads in the lower deck for each rod. Because of this difference, many numbers can be inputted into the Chinese abacus in multiple ways. Find three different ways to input each number in the Chinese abacus and draw the beads' positions:

   **Example:** 5 can be represented as both





   (a) 10

   (b) 25

   (c) 1015