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Grade 6 Math Circles

March 7/8, 2017

Magic and Latin Squares

Solutions

Today we will be solving math and logic puzzles! All these puzzles have unique solutions (meaning there's only one way to solve them)!

Magic Square

A magic square is a square grid in which the numbers of each row, each column, and the main diagonals add up to the same sum! Let's call this sum "S". An example of a magic square is below, find S:

			15
4	9	2	15
3	5	7	15
8	1	6	15
15	15	15	15

Magic squares have been known since 650 BCE and have been used throughout history by magicians and illusionists! Often these numbers would be used to predict the future!

Exercise: Complete these magic squares.

11	13	3
1	9	17
15	5	7

S = 27

48	9	6	39
15	30	33	24
27	18	21	36
12	45	42	3

S = 102

We can also make magic squares with **multiplication** instead of addition where each row, column, and main diagonal multiplies to the same product. Let's call this product P.

Exercise: Complete these multiplicative magic squares.

18	1	12
4	6	9
3	36	2

P = 216

1	15	24	14
12	28	3	5
21	6	10	4
20	2	7	18

P = 5040

Kakuro

Kakuro (also known as mathematical crossword) is a puzzle played on a grid where you need to fill in blank tiles. Here are the rules:

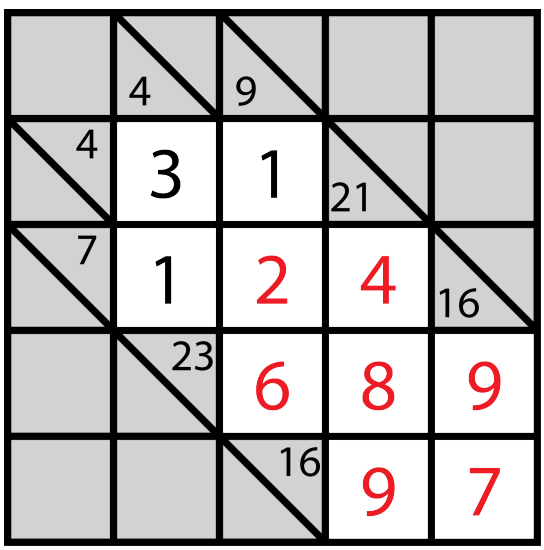
- The tiles are filled in with numbers from 1 to 9.
- The same digit may not appear more than once in one row or column.

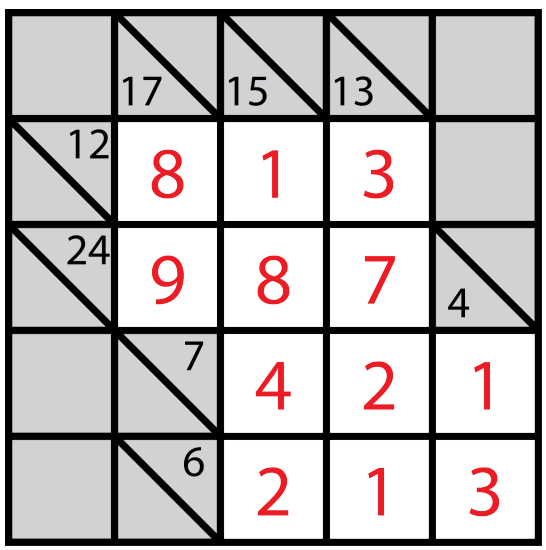
- Tiles with a diagonal line through them tell you how you must fill in the blank tiles! A number in the bottom left of diagonal tiles means that the group of blank tiles **below** must sum to that number. A number in the top right of the diagonal tiles means that the group of blank tiles **to the right** must sum to that number.

Strategies:

- Thinking ahead is very important!
- If you have two or three options for a tile and you're not sure, use a corner and write down a guess lightly. Continuing filling in the corners of other squares and see if your guess works out.
- If you do not know which tile to put a pair of numbers in, use process of elimination by thinking about how your choice would affect other sums.

Exercise: Complete the following Kakuro puzzles.





Latin Square

A Latin square is a square grid (n squares across and n squares down) where each row and each column has the numbers 1 to n. Any number appears in all rows and columns exactly once. There are many different types of logic and math puzzles based on Latin squares!

Sudoku

The most popular type of Latin square is the Sudoku puzzle. This puzzle is on a partially filled out 9×9 grid split into smaller 3×3 blocks. The blank squares must be filled out with the numbers from 1 to 9 such that every smaller block contains each number exactly once and follow all Latin square rules.

Strategies:

- Process of elimination is the key to all Latin square puzzles!
- Look for squares where most of their row, column, or block is already filled out.
- Pick a number that appears often and try to find its occurrence in every row, column, or block.
- If you're really stuck, you can write down a guess in a small corner of a square and continue writing down numbers in the corner of other blank squares to see if your guess is correct. Try to pick a square with only one or two possible numbers for your guess.

Exercise: Complete this sudoku puzzle.

2	4	3	1	7	8	6	5	9
7	8	6	2	5	9	1	4	3
1	5	9	3	4	6	2	7	8
8	1	5	9	2	3	4	6	7
3	6	7	5	1	4	9	8	2
4	9	2	8	6	7	3	1	5
9	3	4	7	8	1	5	2	6
5	7	1	6	3	2	8	9	4
6	2	8	4	9	5	7	3	1

Calcudoku

Calcudoku is a Latin square puzzle where the $n \times n$ grid is divided into sections. The rules are:

- Each section has an operation sign and number in the corner indicating how the numbers in the section relate. (For example, a section with "15×" in the corner means that the numbers multiply to 15.)
- Fill in the blank squares with numbers from 1 to n, following Latin square restrictions.

Strategy:

- Start with smaller sections.
- You may need to take notes.
- Switch between thinking about sections and the rows/columns.

Exercise: Solve these Calcudoku puzzles.

2÷ 4	2	3+ 1	7+ 3
3x 3	1	2	4
2÷ 2	12x 4	3÷ 3	1
1	3	2÷ 4	2

3÷ 3	1	2x 2	1- 4
9+ 4	2	1	3
2÷ 1	3	2÷ 4	2
2	12x 4	3	1

Kropki

Kropki is a Latin square puzzle on an $n \times n$ grid with black and white circles in between the squares. Fill in blank squares with numbers from 1 to n according to these rules:

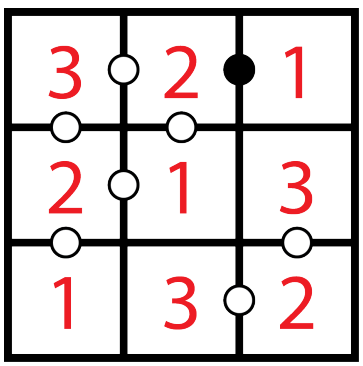
- A black circle means one square is double the other.

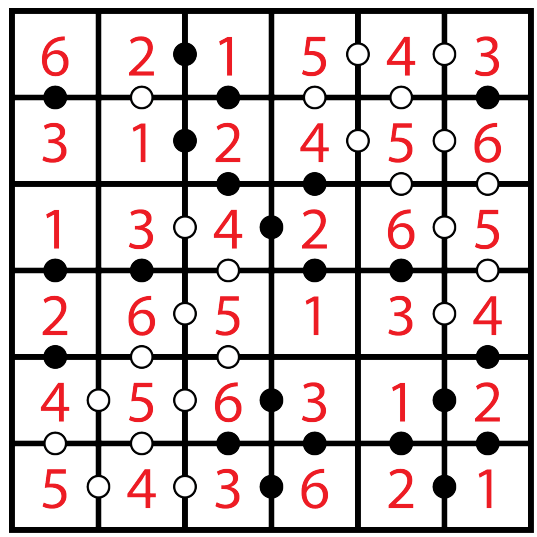
- A white circle means the two squares are consecutive numbers (i.e. 1 and 2 or 4 and 5).
- If there are no circles between two squares, then they are not doubles or consecutives.
- All other Latin square rules apply.

Strategies:

- Look for chains of circles.
- Black circles are often more helpful.

Exercise: Solve the following Kropki puzzles.





Futoshiki

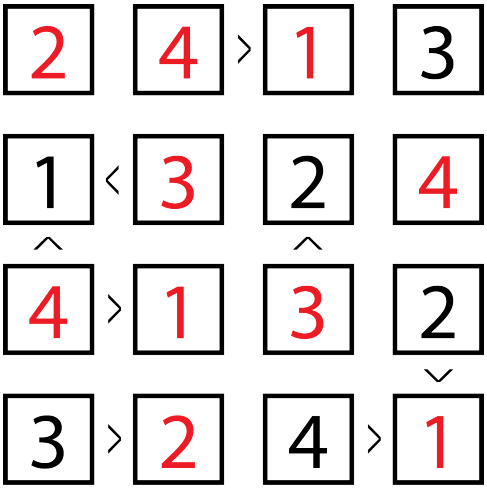
Futoshiki is another Latin square puzzle which uses inequalities. On an $n \times n$ grid with $>$ (greater than) and $<$ (less than) symbols between the squares, fill in the numbers from 1 to n according to these relations and Latin square restrictions.

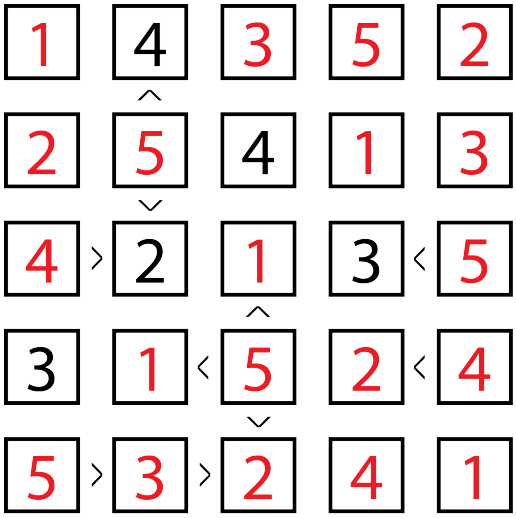
Strategies:

- Look for chains of signs you can follow.
- Use numbers that are already there with a relationship to a blank square.

- Think about where the smallest and largest numbers can or cannot go.
- Look for rows or columns with only one or two blank squares.

Exercise: Complete the following Futoshiki puzzles.





Conclusion

We have looked at 6 different number puzzles which involve different types of logical and mathematical thinking. There are plenty more number puzzles that can you find in the problem set and in puzzle books, apps, and websites. These puzzles can be a fun way to keep your brain active and challenge your logic skills!

Problem Set

1. Which of these are magic squares? If so, what is S?

(a)

10	3	8
5	7	9
6	11	4

Yes. $S = 21$.

(b)

7	2	9
8	5	4
3	10	5

No.

(c)

4	14	15	1
9	17	6	12
5	11	10	8
16	2	8	17

No.

(d)

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Yes. $S = 34$.

2. Complete the following magic squares.

(a) S = 42

11	21	10
13	14	15
18	7	17

(b) S = 195

51	72	3	24	45
69	15	21	42	48
12	18	39	60	66
30	36	57	63	9
33	54	75	6	27

3. Complete the following magic square with only prime numbers where S is unknown. You only know that $S = 120$.

3	61	19	37
43	31	5	41
7	11	73	29
67	17	23	13

4. Make your own 3×3 magic square with $S = 30$.

   Answers will vary. As long as all rows, columns, and main diagonals sum to 30, the answer is correct.

5. Complete these Kakuro puzzles.

(a)

			13	8
		3 / 17	1	2
	10 / 10	1	3	6
24	8	7	9	
11	2	9		

(b)

			16	12
	3	16 / 18	7	9
11	2	5	1	3
16	1	9	6	
	6	4	2	

6. Complete these Sudoku puzzles.

(a)

5	8	1	9	2	4	6	7	3
3	6	7	1	8	5	2	4	9
9	2	4	6	7	3	8	1	5
6	9	2	5	1	7	4	3	8
1	4	8	3	9	6	7	5	2
7	3	5	8	4	2	1	9	6
4	1	9	2	5	8	3	6	7
8	5	3	7	6	1	9	2	4
2	7	6	4	3	9	5	8	1

(b)

6	9	2	7	1	5	4	8	3
8	1	4	6	9	3	7	2	5
3	5	7	2	4	8	1	6	9
4	2	8	3	6	9	5	7	1
5	7	1	8	2	4	9	3	6
9	6	3	5	7	1	2	4	8
1	8	5	4	3	7	6	9	2
2	4	9	1	8	6	3	5	7
7	3	6	9	5	2	8	1	4

7. Odds and Evens is a variant of Sudoku with the added rule that shaded squares contain even numbers and non-shaded squares contain odd numbers.

9	6	8	5	1	3	2	7	4
1	4	2	6	8	7	5	9	3
3	5	7	2	9	4	8	6	1
5	9	3	1	7	2	6	4	8
7	1	6	8	4	5	9	3	2
8	2	4	9	3	6	1	5	7
2	3	5	4	6	8	7	1	9
4	8	9	7	5	1	3	2	6
6	7	1	3	2	9	4	8	5

8. Complete these Calcudoku puzzles.

(a)

2÷ 4	2	2÷ 1	7+ 3
6x 3	4÷ 1	2	4
2	4	3÷ 3	1
1	24x 3	4	2

(b)

24x 4	3	2	5x 1	5
10+ 3	2 2	1- 4	5	1
2	4- 5	12x 1	3 3	2÷ 4
5	1	3	4	2
10+ 1	4	5	1- 2	3

9. Complete these Kropki puzzles.

(a)

2	1	3	4
3	4	2	1
1	2	4	3
4	3	1	2

(b)

3	1	2	5	4
1	4	5	3	2
5	3	4	2	1
2	5	1	4	3
4	2	3	1	5

10. Complete these Futoshiki puzzles.

(a)

5	4	3 >	2	1
2	3	1 <	4 <	5
1	2	4 <	5	3
3	5 >	2 >	1	4
4	1	5	3	2

(b)

5 >	2	4 >	1	3
4	3	2 <	5	1
1	5	3 >	2	4
2	4 >	1	3	5
3 >	1	5 >	4	2

11. Hitori is a puzzle where a square grid is already filled in with numbers and you have to shade out squares to make the grid more like a Latin square. Rules:

    - Shade out squares so that no numbers appear more than once in every row and column.
    - Not every number has to appear in every row and column.
    - Shaded squares cannot be directly connected vertically or horizontally (diagonally does not count).
    - All unshaded squares **must** be connected (diagonally does not count).

    Complete this Hitori puzzle.

3	3	5	2	5
2	4	2	3	5
1	4	4	5	3
4	5	5	1	1
1	3	2	4	1

12. **Challenge problem.** Skyscrapers is a Latin square game where the square grid is the map of a city (from bird's eye view) with each square representing a building. Numbers in the squares represent the height of the building. Numbers outside the grid represent how many buildings a person would see if they were standing in that spot, outside of the city, in the row or column. Remember, if a shorter building is behind a taller one from your point of view, you would not be able to see it. Complete the Skyscraper puzzle:

	3	1	2	2	
2	1	4	2	3	2
3	2	3	4	1	2
1	4	1	3	2	3
2	3	2	1	4	1
	2	3	3	1	