UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles

March 28 & 29, 2017

Math Jeopardy II

Number Theory

$100 Is 37 an emirp?

$200 What is 10000001^3?

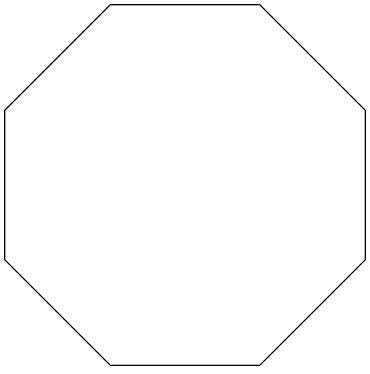
$300 Find the prime factorization of 1470.

$400 How many factors does 245 have?

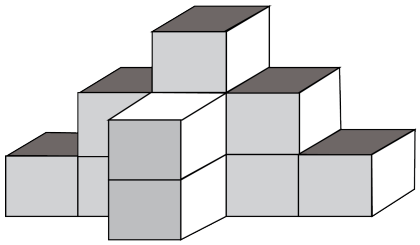
$500 How many integers n are there when $1 \leq n \leq 100$ and n^n is a perfect square.

Spatial Visualization and Origami

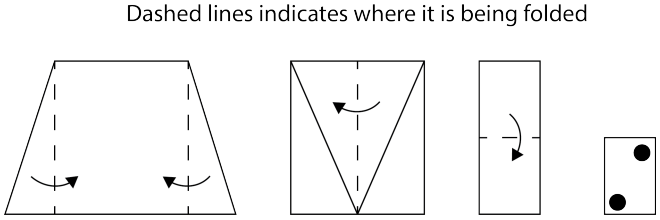
$100 How many lines of symmetry does an octagon have?



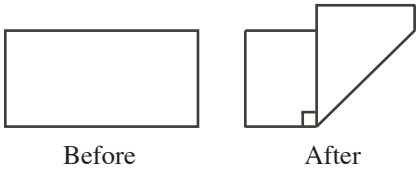
$200 A 3D shape constructed by blocks is shown below. What are the side, top, and front views of the shape?



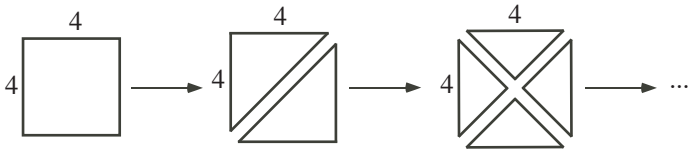
$300 Suppose we fold a figure in the sequence shown below. Suppose we punch two black holes in the last step. What does the figure look like after we unfold it?



$400 A rectangular piece of paper measures 17 cm by 8 cm. It is folded so that a right angle is formed between the two segments of the original bottom edge, as shown. What is the area of the new figure?



$500 A 4×4 square piece of paper is cut into two identical pieces along its diagonal. The resulting triangular pieces of paper are each cut into two identical pieces.



Each of the four resulting pieces is cut into two identical pieces. Each of the eight new resulting pieces is finally cut into two identical pieces. What is the length of the longest edge of the final sixteen pieces of paper?

Fractals

$100 Which of the following does NOT have zero area?

(a) Sierpinski Triangle

(b) Koch Snowflake

(c) Box Fractal

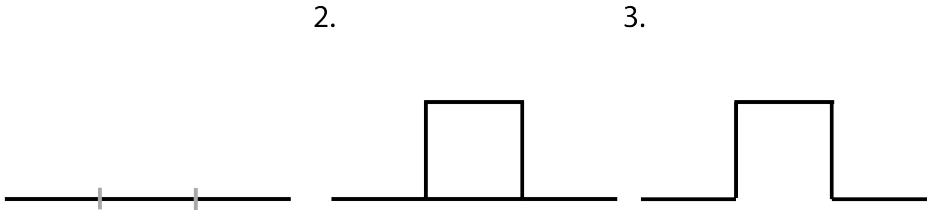
$200 Suppose we have an equilateral triangle with an area of 2 units^2. What is the area of the Sierpinski Triangle after 2 iterations?

$300 Suppose we have an equilateral triangle with side length of 1 unit. What is the perimeter of the Koch Snowflake after 3 iterations?

$400 Here is a variation of the Koch Snowflake.

Suppose we start out with a straight line with a length of 9 units.

1. Divide the line into 3 equal line segments
2. In the middle section, using that side length, draw a square with the middle section as one of the sides of the square.
3. Erase the middle section. We are done the first iteration.

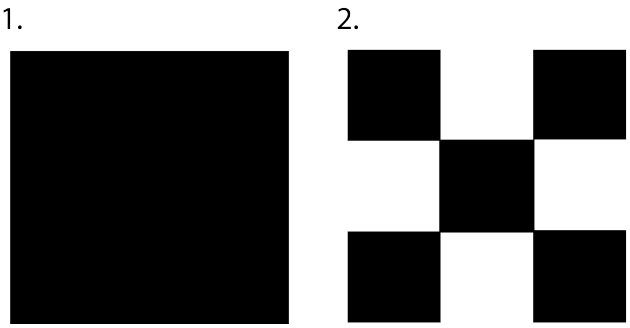


4. Repeat steps 1-3 for every line you see for each iteration.

Now, what does the third iteration looks like? What is the total length of the line?
Hint: You could derive a formula, but it isn't necessary in this case.

$500 Suppose we have a black square with an area of 27 units^2.

1. Divide the black square into 3 by 3 grid i.e. you should have 9 equal smaller squares within the initial black square.
2. Shade the middle squares on the outside white i.e. you should have 4 white squares as shown below. We are done our first iteration.



3. Repeat steps 1-2 for each black square for subsequent iterations.

Derive a formula to determine the area of all the black squares after the n^{th} iteration.

Sets

$100 What is the cardinality of the set $\{12, 3, 4\}$?

$200 How many subsets can you form from a set with 3 elements?

$300 Given a set with $A = \{2, 3, 6\}$ and $B = \{5, 6, 2, 3\}$ and $\mathbb{U} = \{1, 2, 3, 4, 5, 6\}$. Determine $\overline{\bar{A} \cup B}$.

$400 There are 120 grade 7/8 math circles registered across the 4 sections on Tuesday and Wednesday nights. During registration, 74 students preferred to be part of the Tuesday section and 65 students wanted to be part of the Wednesday section. How many students didn't care on whether they were in the Tuesday or Wednesday section?

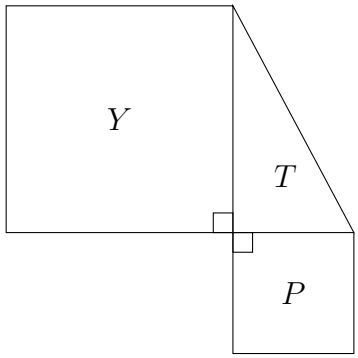
$500 Recall that a number is a perfect square if it can be expressed as a product of two of the same integers. Let $P = \{1, 4, 9, 16...\}$ be the set of perfect squares. Let $O = \{1, 3, 5, 7, ...\}$ i.e. the set of all odd natural numbers. Show using a rule that the two sets have the same cardinality.

Random Questions I

\$100 Hermela doubled a number and then added 4 to get 42. If she had first added 4 to the original number and then doubled the result, what would the answer have been?

\$200 What is the smallest positive integer by which 54 can be multiplied so that the result is a perfect square?

\$300 The perimeter of square Y is 60 cm and the perimeter of square P is 32 cm. What is the perimeter of right-angled triangle T?



\$400 An integer is chosen at random from 1 to 60 inclusive. What is the probability the integer chosen contains the digit 4?

\$500 Find the value of $490 - 491 + 492 - 493 + 494 - 495 + \ldots - 509 + 510$.

Gauss Prep

\$100 If $x = -4$ and $y = 4$, which of the following expressions gives the largest answer?

(a) $\frac{x}{y}$ (b) $y - 1$ (c) $x - 1$ (d) $-xy$ (e) $x + y$

\$200 If $2^a = 8$ and $a = 3c$, then c equals

(a) 0 (b) $\frac{3}{4}$ (c) 1 (d) $\frac{4}{3}$ (e) 6

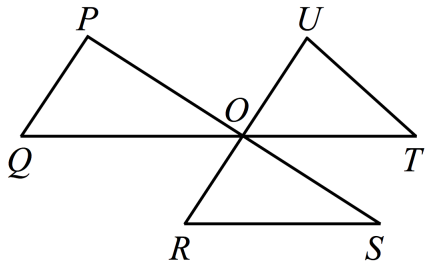
$300 Last year, Kiril's age was a multiple of 7. This year, Kiril's age is a multiple of 5. In how many years will Kiril be 26 years old?

(a) 11 (b) 21 (c) 4 (d) 18 (e) 16

$400 Andrea has finished the third day of a six-day canoe trip. If she has completed $\frac{3}{7}$ of the trip's total distance of 168 km, how many km per day must she average for the remainder of her trip?

(a) 29 (b) 24 (c) 27 (d) 32 (e) 26

$500 Lines PS, QT and RU intersect at a common point O,as shown. P is joined to Q, R to S, and T to U, to form triangles. The value of $\angle P + \angle Q + \angle R + \angle S + \angle T + \angle U$ is



(a) 450° (b) 270° (c) 360° (d) 540° (e) 720°

Game Theory

$200 What is the equilibrium of the game?

Player 1		Player 2: X	Player 2: Y
	A	(10, 10)	(15, 5)
	B	(5, 15)	(12, 12)

$400 How many equilibria (in pure strategies) does the game below have?

Player 1		Player 2: Rock	Player 2: Paper	Player 2: Scissors
	Rock	(0, 0)	(−1, 1)	(1, −1)
	Paper	(1, −1)	(0, 0)	(−1, 1)
	Scissors	(−1, 1)	(1, −1)	(0, 0)

$600 **Game of Chicken** Once a breakthrough is made by any one company, it quickly becomes an open standard for other companies to adopt. As a result, companies can benefit from other companies investing or researching into emerging technologies because they themselves don't have to. Suppose Apple and Samsung both want to make a product that introduces a breakthrough into artificial intelligence.

1. If both Apple and Samsung invests into A.I, they will both earn 300 million dollars annually .
2. If ONLY Apple invests into A.I, Apple and Samsung will earn 500 million dollars and 100 million dollars per annum respectively.
3. If ONLY Samsung invests into A.I, Apple and Samsung will earn 100 million dollars and 500 million dollars per annum respectively.
4. If neither invests, both companies will not earn anything.

Draw the payoff matrix of the situation and what is the Nash Equilibrium?

$800 Kingeon and Caila decided to fundraise for a roadtrip. They can either sell bags of chips or cans of pop.

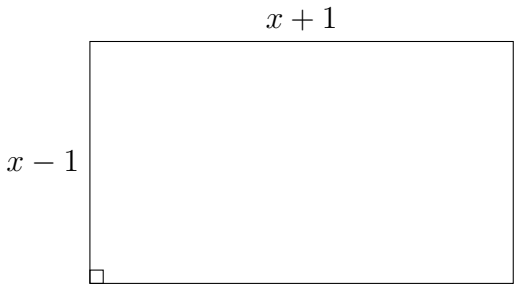
1. If Kingeon and Caila both sell cans of pop, then Kingeon will sell 35 cans of pop and Caila will sell 50 cans of pop.
2. If Kingeon and Caila both sell bags of chips, then Kingeon will sell 30 bags of chips and Caila will sell 50 bags of chips.
3. If Kingeon sells cans of pop and Caila sells bags of chips, then Kingeon will sell 50 cans of pop and Caila will sell 30 bags of chips.
4. If Kingeon sells chips and Caila sells cans of pop, then Kingeon will sell 30 bags of chips and Caila will sell 70 cans of pop.

Is this a fair competition? If not, what can Kingeon do to ensure that their fundraiser is as successful as possible?

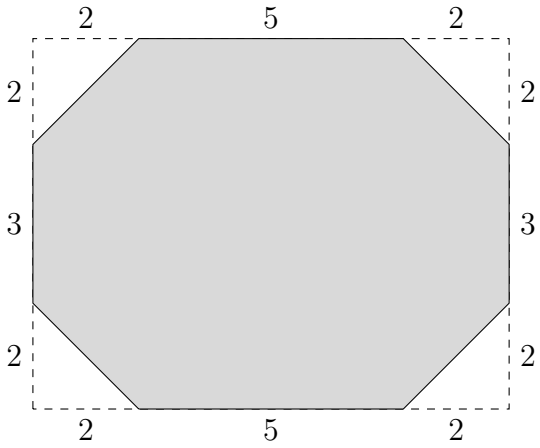
$1000 Natasha and Steve are playing a game of 6. In this game, there are 6 sticks in a pile. Each player can take away 1, 2 or 5 sticks on their turn. Whichever player picks up the last stick wins the game. If Natasha is the first player, who will win the game?

Geometry

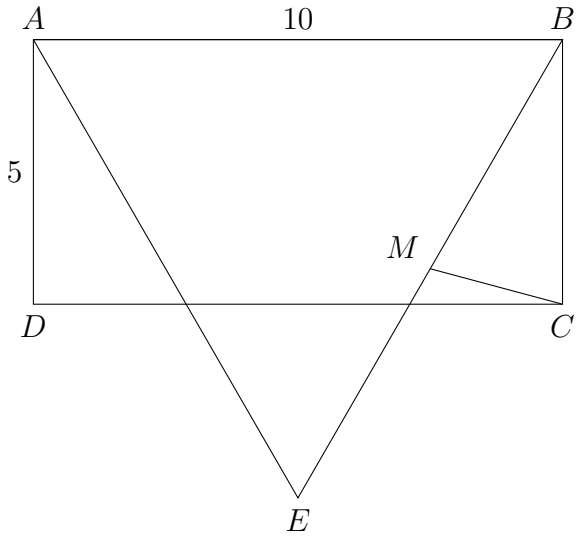
$200 If the perimeter of the given rectangle is 24, what is the value of x?



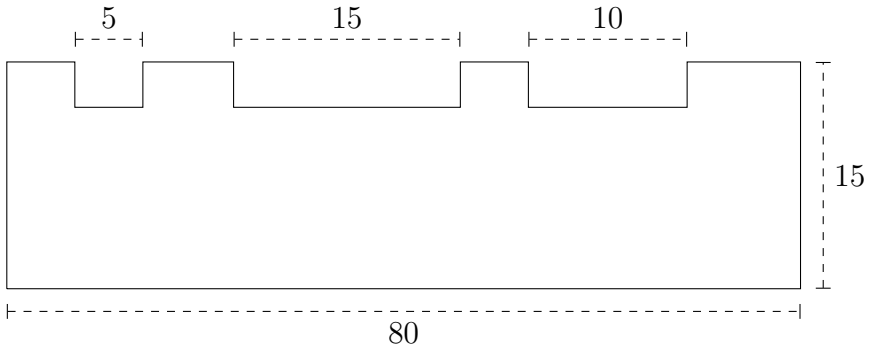
$400 In the 9 by 7 rectangle shown, what is the area of the shaded region?



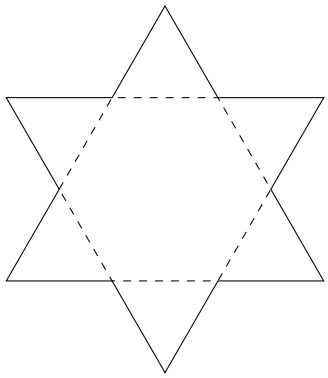
$600 In the diagram, $ABCD$ is a rectangle and ABE is an equilateral triangle. M is the midpoint of BE. Calculate $\angle BMC$.



$800 Three small rectangles, of the same depth, are cut from a rectangular sheet of metal. The area of the remaining piece is 990. What is the depth of each cut?



$1000 A six-pointed star is formed by extending the edges of a regular hexagon. If the perimeter of the hexagon is 21, what is the perimeter of the star?



Combinatorial Counting

$200 A fog horn sounds a blast for 2 seconds and then is silent for 8 seconds. How many blasts are made in a $3\frac{1}{2}$ hour period?

$400 In a class of 30 students, twelve played in the band, and seventeen played volleyball. Of these, five students did both. How many students did not participate in either activity?

$600 In how many ways can we arrange the letters in MARCH?

$800 In how many ways can we arrange the letters in BANANA?

$1000 Solve for n in the following: ${}_nC_2 = 66$

Logic

$200 If Monica's daughter is my daughter's mother, who am I to Monica?

$400 At a family reunion of 12 people, the official photographer takes pictures of two people at a time. If each person has his picture taken with each of the other people, determine the maximum number of pictures that could be taken.

$600 There is a box full of fruits.
All but two are mangoes.
All but two are apples.
All but two are oranges.
How many fruits are in the box?

$800 Using any math symbols, arrange four 9s so that they equal 100.

$1000 Ana, Bernie, Carmelo, Daniel and Elle are all in the same house.

1. If Ana is watching a movie, so is Bernie.
2. Either Daniel, Elle, or both of them are watching a movie.
3. Either Bernie or Carmelo, but not both are watching a movie.
4. Daniel and Carmelo are either both watching or both not watching a movie.
5. If Elle is watching a movie, then Ana and Daniel are also watching a movie.

Who is watching a movie and who is not?

Random Questions II

$200 Which of the following numbers are greater than its square?

$$-20, -\frac{3}{4}, 0, \frac{1}{2}, 10$$

$400 My rich uncle gave me $1 for my first birthday. On each birthday after that, he doubled his previous gift. Find the total amount that he had given me by the day after my 8^{th} birthday.

$600 There are 15 Blue Jays and 14 Orioles perched in 3 trees. Each tree has at least 4 Blue Jays and 2 Orioles. If no tree has more Orioles than Blue Jays, then what is the largest number of birds that can be in one tree?

$800 How many numbers in the set {10, 11, 12, ..., 99} increase in value when the order of their digits is reversed?

$1000 Given $xy = \frac{x}{y} = x - y$, where $y \neq 0$, determine the value of $x + y$.

Gauss Prep

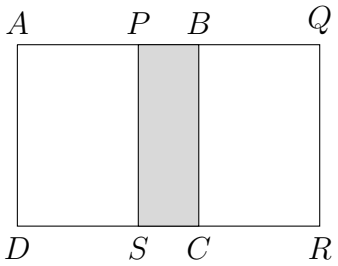
$200 The value of $(1 + 2)^2 - (1^2 + 2^2)$ is

(a) 14 (b) 4 (c) 2 (d) 12 (e) 1

$400 If the mean (average) of five consecutive integers is 21, the smallest of the five integers is

(a) 17 (b) 21 (c) 1 (d) 18 (e) 19

$600 Two identical squares, $ABCD$ and $PQRS$, have side length 12. They overlap to form the 12 by 20 rectangle $AQRD$ shown. What is the area of the shaded rectangle $PBCS$?

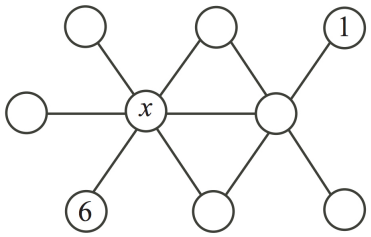


(a) 24 (b) 36 (c) 48 (d) 72 (e) 96

$800 At the Gaussland Olympics, there are 480 student participants. Each student is participating in 4 different events. Each event has 20 students participating and is supervised by 1 adult coach. There are 16 adult coaches and each coach supervises the same number of events. How many events does each coach supervise?

(a) 12 (b) 8 (c) 6 (d) 16 (e) 15

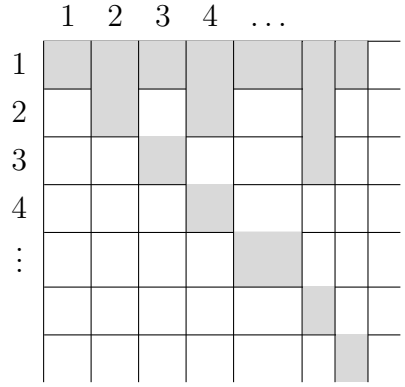
$1000 In the diagram, each of the integers 1 through 9 is to be placed in one circle so that the integers in every straight row of three joined circles add to 18. The 6 and 1 have been filled in. The value of the number represented by x is



(a) 4 (b) 5 (c) 8 (d) 7 (e) 3

Final Jeopardy

In the diagram, the grid has 150 rows and 150 columns, numbered from 1 to 150. In row 1, every box is shaded. In row 2, every second box is shaded. In row 3, every third box is shaded. The shading continues in this way, so that every n-th box in row n is shaded. Which *column* has the greatest number of shaded boxes?



(a) 144 (b) 120 (c) 150 (d) 96 (e) 100