



Intermediate Math Circles

November 15, 2017

Probability III

Problem Set Solutions

Problem Set

1. If you take a bus to work in the morning, there is a 30% chance that you will arrive late. When you go by bicycle there is a 10% chance you will arrive late. 70% of the time you go by bike, and 30% of the time you go by bus. Given that you arrive late, what is the probability that you took the bus?

Solution:

Let B represent that you take the bus, and let L represent that you are late. Then

$$\begin{aligned}P(B|L) &= \frac{P(L|B)P(B)}{P(L|B)P(B) + P(L|B^c)P(B^c)} \\ &= \frac{(0.3)(0.2)}{(0.3)(0.2) + (0.1)(0.8)} \\ &= \frac{3}{7}\end{aligned}$$

2. A box contains 4 coins – 3 fair coins and 1 biased coin for which $P(\text{heads}) = 0.6$. A coin is picked at random and tossed 5 times. It shows 3 heads. Find the probability that the coin is fair.

Solution:

Let F represent that the coin chosen is fair and let H represent that 3 heads were shown. Then

$$\begin{aligned}P(F|H) &= \frac{P(H|F)P(F)}{P(H|F)P(F) + P(H|F^c)P(F^c)} \\ &= \frac{\binom{5}{3} \left(\frac{1}{2}\right)^5 \times \left(\frac{3}{4}\right)}{\binom{5}{3} \left(\frac{1}{2}\right)^5 \times \left(\frac{3}{4}\right) + \binom{5}{3} (0.6)^3 (0.4)^2 \times \left(\frac{1}{4}\right)} \\ &= 0.73065\end{aligned}$$



3. At a police spot check, 10% of cars stopped have defective break lights and a faulty muffler. 15% of the cars have a defective break lights, but the muffler was ok. If a car is stopped and has defective break lights, what is the probability that the muffler is also faulty?

Solution:

Let B represent the event that a break lights are defective and let M represent the event that a muffler is defective.

$$\begin{aligned} P(M|H) &= \frac{P(M \cap H)}{P(H)} \\ &= \frac{P(M \cap H)}{P(M \cap H) \cup P(M^c \cap H)} \\ &= \frac{0.1}{0.1 + 0.15} \\ &= 0.4 \end{aligned}$$

4. For one deck building project, a contractor buys 70% of the lumber from supplier A, and 30% of the lumber from supplier B. Typically, 85% of the lumber from supplier A arrived undamaged, while 90% of the lumber from supplier B arrived undamaged. If the contractor pulls one board out and its damaged, what is the probability that the damaged board is from

(a) Supplier A?

Solution:

Let A represent supplier A and D represent that wood is damaged.

$$\begin{aligned} P(A|D) &= \frac{P(D|A)P(A)}{P(D|A)P(A) + P(D|A^c)P(A^c)} \\ &= \frac{(0.15)(0.7)}{(0.15)(0.7) + (0.1)(0.3)} \\ &= \frac{0.105}{0.135} \\ &= 0.778 \end{aligned}$$

(b) Supplier B?

Solution:

Let B represent supplier B and D represent that wood is damaged.

$$\begin{aligned} P(B|D) &= \frac{P(D|B)P(B)}{P(D|B)P(B) + P(D|B^c)P(B^c)} \\ &= \frac{(0.1)(0.3)}{(0.1)(0.3) + (0.15)(0.7)} \\ &= \frac{0.03}{0.135} \\ &= 0.222 \end{aligned}$$