



Intermediate Math Circles

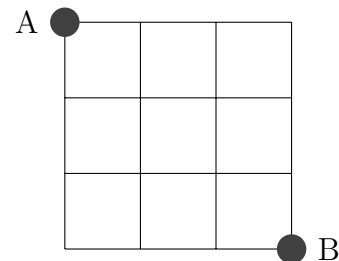
November 1, 2017

Probability I

Problem Set

- Suppose we draw one card from a well-shuffled deck. Let A be the event that we get a spade, and B be the event we get an ace.
 - Are these events mutually exclusive?
 - What is the probability of drawing an ace or a spade?
- Mark has a bag that contains 3 black marbles, 6 gold marbles, 2 purple marbles, and 6 red marbles. Mark adds a number of white marbles to the bag and tells Susan that if she now draws a marble at random from the bag, the probability of it being black or gold is $\frac{3}{7}$. How many white marbles did Mark add to the bag?
- In the 6/49 lottery, six different numbers must be selected between 1 through 49 inclusive (order is not important).
 - To win the grand prize your 6 numbers must match the 6 numbers drawn. What is the probability of winning the grand prize?
 - To win any prize you must have at least 2 numbers that match the 6 numbers drawn. What is the probability of winning any prize?
- What is the probability of being dealt 1 king in a 4 card hand?
 - Write a formula for the probability of being dealt exactly i kings in a 4 card hand.
- Suppose a die is weighted so that when it is rolled, the probability of seeing any number on the top face is proportional to the number on the face.
 - Give the probability distribution that would apply.
 - What is the probability of rolling a multiple of 3?
- A network of paths forms a grid as shown in the following diagram.

Abby starts at point A and walks towards point B . At the same time Bruno starts at point B and walks towards point A . Neither person follows a particular route, but they are always moving towards their destination. What is the probability that they will meet if they both walk at the same rate?





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Probability I Problem Set Answers

- (a) No, these events are not mutually exclusive.

(b) The probability of drawing an ace or a spade is $\frac{4}{13}$.
- Mark added 4 white marbles to the bag.
- a) The probability of winning the grand prize is $\frac{1}{13\,983\,816}$.

b) The probability of winning any prize is 15.1%.
- (a) $P(\text{exactly 1 king}) = \frac{\binom{4}{1}\binom{48}{3}}{\binom{52}{4}} = \frac{69\,184}{270\,725} \doteq 0.256$

(b) $P(\text{exactly } i \text{ kings}) = \frac{\binom{4}{i}\binom{48}{4-i}}{\binom{52}{4}}, \text{ for } i = 0, 1, 2, 3, 4$
- (a) The following probability distribution would apply.
 $p_1 = \frac{1}{21}, p_2 = \frac{2}{21}, p_3 = \frac{3}{21}, p_4 = \frac{4}{21}, p_5 = \frac{5}{21}, p_6 = \frac{6}{21}$

(b) The probability of rolling a multiple of 3 is $\frac{3}{7}$.
- The probability that Abby and Bruno meet is $\frac{5}{16}$.