Population Models: Week 2

Question 1 Consider the population model for pairs of goats on an island, with hunters as discussed in the slides

$$P^{n+1} = \lfloor (1 + \alpha - \beta)P^n \rfloor - H^n$$

Set $\alpha = 1$ and $\beta = 0.9$. If $P^0 = 100$ compute a table of P^n for $n = 1, ..., 5$ with 2, 4 and 6 hunters. Then get together with three of your neighbours and compare what you got. Did all of you get the same result? What is a sensible way to put all the results together?

Question 2 If $\alpha = \beta$ what does the hunting model predict (you can do this for an arbitrary of hunters or if you'd prefer 5 hunters)? Is there a way to make the prediction more quantitative (HINT: think about what happens on average). Recall that the average of 5 random events is the sum of all 5 values divided by 5. Similarly for N events it is a sum of all the events divided by N.

Question 3 In the last question what you actually showed is that in a population that undergoes neither net birth or net death, hunting leads to inevitable extinction. Now imagine an alternative scenario in which each year instead of 5 hunters, 5 conservationists visit our island and flip a coin to see if they release a pair of goats or take it back home with them (a little weird I know). Write down the model for this scenario. If $\beta = 1$, $\alpha = 0.95$ carry out an experiment for 5 successive years and again compare with 4 neighbours.

Question 4 If we now have a model with 5 hunters (like Question 1) and 5 conservationists (like Question 3) and $\alpha = \beta$ can you show that on average the population stays the same? Do an experiment for 5 years. Does the population stay the same in your experiment? Unless you are particularly lucky, you should find it does not. Can you explain why not? HINT: the model can be written down as:

$$P^{n+1} = \lfloor (1 + \alpha - \beta)P^n \rfloor - H^n + C^n$$

where H^n is the random part due to hunters and C^n the random part due to conservationists.

Question 5 If we have $\beta = 1$ and $\alpha = 0.8$ explain how you would find the number of conservationists needed to ensure that on average the population does not go down. Note: this is a difficult problem that you can consider as a challenge for after the session.