



# Intermediate Math Circles

## October 25, 2017

### Problem Set - Counting III

1. How many permutations can be formed from the letters of “MNEMONIC”.
2. A bit string is a sequence in which each term is either 0 or 1. For example, 0001110100 is a bit string of length 10 with six 0’s and four 1’s. How many bit strings of length 10 can be made with six 0’s and four 1’s if:
  - a) there are no restrictions.
  - b) the bit string must begin and end with a zero.
  - c) the 0’s must be in pairs and the 1’s must be in pairs. For example, 0011000011 would be such a string. Notice that 2 pairs of pairs of zeros can be together.
3. At this time of the year many people string lights across the front of their homes. A certain home owner had a string of lights with 24 spaces for bulbs. He had 6 red bulbs, 6 green bulbs, 6 yellow bulbs and 6 blue bulbs. How many different ways can the home owner put all of the bulbs in the sockets if:
  - a) there are no restrictions?
  - b) a red bulb must always be in a position to the immediate right of a green bulb as he looked from left to right?
  - c) all lights of the same colour must be together?
4. Using all of the letters of the name *Mississippi*, how many permutations can be formed if:
  - a) there are no restrictions?
  - b) the permutation must begin with *M*?
  - c) the two *p*’s must be together?
  - d) the two *p*’s must never be together?
  - e) the permutation must be a palindrome? That is, the permutation must read the same when read frontwards and backwards. *a toyota* is an example of a palindrome (the space is not considered part of the permutation.).
5. Anna Lize has seven loonies and wants to distribute them among her three friends. In how many ways can she do this if:
  - a) it is acceptable for one or more of her friends get nothing.
  - b) each friends gets at least 1 loonie.
  - c) her best friend, Ann Alysis, gets at least half of the coins and the other 2 friends gets at least 1 loonie.

Answers can be found on the back side of this sheet.. If you disagree with an answer, try again. Full solutions will be posted on our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) by the end of this week.





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**Answers Only**

1. There are 10 080 permutations of the letters of *MNEMONIC*.
2.
  - a) There are 210 bit strings of length 10.
  - b) There are 70 bit strings of length 10 that begin and end with zero.
  - c) There are 10 bit strings of length 10 in which the zeroes are in pairs and the ones are in pairs.
3.
  - a.) There are  $\frac{24!}{6! \times 6! \times 6! \times 6!}$  arrangements of the 24 lights.
  - b.) There are  $\frac{18!}{6! \times 6! \times 6!} = 17\,153\,156$  arrangements of the bulbs.
  - c.) There are 24 ways to arrange the bulbs.
4.
  - a) There are 34 560 arrangements with no restrictions.
  - b) There are 3 150 arrangements beginning with *M*.
  - c) There are 1 575 arrangements in which the two *p*'s are together.
  - d) There are 33 075 arrangements in which the two *p*'s are not together.
  - e) There are 30 arrangements which are palindromes.
5.
  - a.) There are 36 ways to distribute the 7 loonies.
  - b.) There are 15 ways to distribute the 7 loonies if each friend must get at least one loonie.
  - c.) There are 3 ways to distribute the coins.

