UNIVERSITY OF WATERLOO
Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
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Grade 7/8 Math Circles

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Scientific Equations

Introduction

In *any* equation in mathematics, physics, chemistry, economics, or *anywhere* else where equations might appear, there will almost always be *three* things which you will find in these equations.

The first type of thing you might see in an equation is called a 'constant'. An example of an equation with a constant in it is:

$$Area = \pi \times r^2$$

The constant in this equation is π. It is called a constant because it is a number that will always have the same value. In other words, it is a *constant* value.

The next thing which you might encounter in an equation is known as a 'variable'.

$$Area = Length \times Width$$

The variables in this equation are $Area$, $Length$, and $Width$.

They are called variables because they are numbers that can have different values.

The last thing which you will see in an equation is called an 'operator'.

$$A + 3 = (9 \times x) \div 2$$

The operators in this equation are $+, =, (), \times, \div$.

Operators are the things that tell you what to do with constants and variables.

Making your own equation

It probably sounds tricky to make your own equation, but it might be simpler than you think. In Example 1, we will make our own equation which will tell us about how a sloth travels!

Example 1:

Josh the sloth is bolting across the jungle floor at a constant speed of 0.2 kilometers per hour.

(a) What distance will Josh have travelled if:

(i) 3 hours have passed? 0.6 kilometers

(ii) 7.5 hours have passed? 1.5 kilometers

(iii) 15 minutes have passed? 0.05 kilometers (Be careful, 15 minutes is 0.25 hours!)

(b) What steps did you follow to solve for the distance travelled in each of the previous questions? How far will Josh travel if *'T'* hours have passed?

I multiplied the speed by the time in each of the previous questions, so I will do the exact same in this question. Josh the sloth will travel a distance of $0.2 \times T$ kilometers!

(c) Write an equation which you can use to calculate the distance *'D'* which Josh will travel after *'T'* hours.

$$Distance = 0.2\,\frac{km}{hour} \times Time \qquad \text{Or} \qquad D = 0.2 \times T$$

(d) What constants, variables, and operators are there in this equation?

Constants: 0.2 Variables: D and T Operators: $\times$ and $=$

The equation that we found in this example is one variation of one of the most widely used equations in the entire world. This is the same equation which is used to calculate how long a flight will take, how fast olympic sprinters run, and how far quarterbacks can throw a football.

The next thing we will talk about is how to use this equation, or other equations like this, to solve many many different problems in a variety of different areas.

Using the Equation

In Example 1, we found out that Josh the sloth travels a distance which can be modeled by this equation:

$$D = 0.2 \times T$$

For a different animal, or a vehicle, or something else that travels at a different speed, say 'S', this equation will become:

$$D = S \times T \qquad \text{OR} \qquad Distance = Speed \times Time$$

It is very common to use only the first letter of a variable when writing an equation (instead of '$Distance$' we use 'D'). This is just because it saves space and makes it easier to write.

So now we have an equation which we can use to calculate the distance that *anything* will travel, when it moves at *any* speed, for *any* amount of time.

Example 2:

(a) Sophie's bus ride to school each morning takes about half an hour. Judging from the speedometer, and from the amount of time the bus spends at stoplights, Sophie assumes that the average speed the bus travels at is 30 kilometers per hour. How far does Sophie calculate her trip to school is?

$D = S \times T = 30\,\frac{km}{hour} \times 0.5\,hours = 15\,kilometers$

(b) A cheetah can run at a top speed of 120 kilometers per hour. If it can only maintain this speed for 1 minute before having to stop and rest, how far will it have travelled?

$D = S \times T = 120\,\frac{km}{hour} \times \frac{1}{60}\,hours = 2\,kilometers$

(c) ***Think carefully:*** You are leaving from Toronto and heading to Vancouver on a plane that travels at 500 kilometers per hour. When you leave Toronto, the clock at the airport reads 9:30 AM. Once you arrive in Vancouver, the clock at that airport reads 3:30 PM. What is the distance between Vancouver and Toronto?

The trick to this question is to notice that due to time zones, the time in Vancouver will be 3 hours behind the time in Toronto. Therefore:

$$D = S \times T = 500\,\frac{km}{hour} \times (6\,hours + 3\,extra\,hours) = 4,500\,kilometers$$

Rearranging the Equation

This equation does not only have one purpose. We can use it to solve for the distance travelled, but we can also use it to solve for both the speed, and the time taken!

Here is how we can use our distance equation to solve for speed!

1. Write out your distance equation.

$$D = S \times T$$

2. We want to get an equation for the speed *'S'*, so what we have to do is get *'S'* alone by itself on one side of the equation (i.e. S = stuff). This is known as 'isolating' a variable.

   Right now, *'T'* is on the same side of the equals sign as *'S'* so we have to get rid of it. Because *'T'* is being multiplied onto *'S'*, we have to do something that can get rid of the multiplication. Division is the opposite of multiplication, so if we divide by *'T'*, it will cancel out the multiply by *'T'*!

$$\frac{D}{T} = \frac{S \times T}{T}$$

   When we divided by *'T'* above, we had to do it on both sides of the equals sign. If we only divided on one side of the equals sign, then the two sides would *not* be equal to each other anymore.

3. The last thing to do now is simplify our equation.

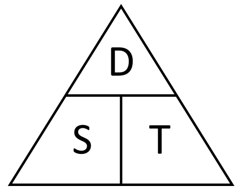
$$\frac{D}{T} = \frac{S \times T}{T}$$
$$\frac{D}{T} = S \times \frac{T}{T}$$
$$\frac{D}{T} = S$$

   So *Speed* can be calculated by dividing *Distance* by *Time*.

What equation can we use to solve for the time *'T'*?

$$D = S \times T$$
$$\frac{D}{S} = \frac{S \times T}{S}$$
$$\frac{D}{S} = \frac{S}{S} \times T$$
$$\frac{D}{S} = T$$

In the beginning, we started with just a distance equation, but now we have equations for distance, speed, and time! One way of remembering these equations is to write the three variables in a triangle like this:



To use this triangle, first decide which variable you want to know the equation for, and then look at how the other two variables are positioned.

If you want to know what the equation for *'D'* is, you have to look at the other two variables *'S'* and *'T'*. Because they are beside each other, it means they are multiplied by each other.

$$D = S \times T$$

If you want to know what the equation for *'T'* is, you have to look at the other two variables *'D'* and *'S'*. Because *'D'* is on top of *'S'*, it means *'D'* is being divided by *'S'*.

$$T = \frac{D}{S}$$

Similarly, if you looking for the equation for *'S'*, you will see that it has to be *'D'* divided by *'T'*.

$$S = \frac{D}{T}$$

Example 3:

(a) The fastest recorded time to run the 100 meter dash is 9.58 seconds by Usain Bolt. How fast did Usain run on average to attain this time?

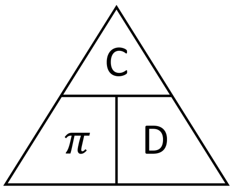
$S = \frac{D}{T} = \frac{100}{9.58} = 10.44\ \frac{meters}{second}\ or\ 37.6\ \frac{km}{hour}$

(b) The distance to Mars from Earth is about 54,600,000 kilometers. If a modern day spacecraft can travel 227,500 kilometers in a single day, how many days would it take to get to Mars?

$T = \frac{D}{S} = \frac{54{,}600{,}000}{227{,}500} = 240\ days$

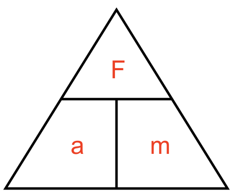
Example 4:

1. What are the three equations that this triangle represents?



$C = \pi \times D$ $\pi = \frac{C}{D}$ $D = \frac{C}{\pi}$

2. Write the equation below so it properly fits inside of the triangle, then write the other two variations of the equation.



$a = \frac{F}{m}$ $m = \frac{F}{a}$ $F = m \times a$

Other Equations

At the very beginning of this lesson, it was mentioned that you can find equations like these all over the place. What we will do now is explore some of these equations, and see what different kinds of questions we can solve with each of them.

Chemistry

Chemistry is the study of matter (anything with a mass and a volume), and how different kinds of matter interact with one another. One of the most important properties of matter, which can help determine what these interactions will be, is known as *density*.

What density does is tell us how heavy something is. For example, if you fill one bucket with water, and another bucket of the same size with dirt, the bucket full of dirt will be heavier because dirt has a higher density than water!

How can we find an equation for density? If we can say that a bucket of dirt is *heavier* than a bucket of water, when *both* buckets are the *same size*, then that must mean that the dirt is heavier *per* amount of volume.

To summarize, if one substance is *denser* than another, that means it must have more mass per some amount of volume. In other words, density is the amount of mass per volume that a substance has, or if written in math terms:

$$Density = \frac{Mass}{Volume}$$

Physics

Physics is the study of *all* phenomena in the entire universe. A few examples of things studied in physics are heat, energy, forces, power, time, and much more.

You have already seen equations for two of these from earlier:

$$T = \frac{D}{S} \qquad \longrightarrow \qquad Time = \frac{Distance}{Speed}$$

$$F = m \times a \qquad \longrightarrow \qquad Force = mass \times acceleration$$

Another important equation in physics is actually a combination of the *Force* equation and the *Distance* equation.

$$E = F \times D \qquad \longrightarrow \qquad Energy = Force \times Distance$$

The unit that physicists use to measure *Force* is known as the 'Newton'. The unit that physicists use to measure *Energy* is known as the 'Joule'.

To use 1 Joule of energy, you must apply a force of 1 Newton across a distance of 1 meter.

$$Energy = Force \times Distance$$

$$Energy = 1\, Newton \times 1\, meter$$

$$Energy = 1\, Joule$$

A unit of energy that you might be more familiar with is a Calorie.

4184 Joules = 1 Calorie

Economics

Whenever you are running any kind of business, there are always two things which are going to be very important to you. Revenue and Profit.

Revenue is simply the *total* amount of money which a business generates. The more of a product which a business sells, and the higher a price they sell it at, are the two features which determine how much revenue a business can produce. This tells us the equation for revenue must be:

$$Revenue = Quantity \times Price$$

Profit on the other hand is how much money a business has made after paying all of their expenses. For example, if a lemonade stand sells a single glass of lemonade for \$1.00, they will have made \$1.00 of *revenue*. If the water, sugar and lemons used to make that glass of lemonade cost \$0.25, that means they would have only made \$0.75 of *profit*.

$$Profit = Revenue - Total\ Expenses$$

Example 5:

1. A 150 gram silver necklace contains 14.3cm^3 of silver. A 50 gram gold ring contains 2.56cm^3 of gold. Which material is more dense, silver or gold? How many times denser is it than the other material?

$D_{silver} = \frac{m}{V} = \frac{150}{14.3} = 10.5\ \frac{grams}{cm^3}$ $D_{gold} = \frac{m}{V} = \frac{50}{2.56} = 19.5\ \frac{grams}{cm^3} = 1.86 \times D_{silver}$

2. Mark the moving man was hired to move a piano from Chopin's house to the Royal Theatre. If the distance between the two locations is 500 meters, and Mark needs to apply a force of 2,000 Newtons to move the piano, how many Calories will Mark burn?

$E = F \times D = 2,000\ Newtons \times 500\ meters = 1,000,000\ Joules = 239\ Calories$

3. Starbucks sells around 3,800,000,000 cups of coffee per year, at around \$2.10 per cup. If it costs them \$1.25 overall to produce each cup of coffee, how much profit does Starbucks make each year?

$Revenue = Quantity \times Price = 3,800,000,000 \times \$2.10 = \$7,980,000,000$

$Profit = Revenue - Total\ Expenses = \$7,980,000,000 - (3,800,000,000 \times \$1.25)$
$= \$7,980,000,000 - \$4,750,000,000 = \$3,230,000,000$

Unit Conversions

Before you begin the problem set, there are a few things you should know about unit conversions.

Unit conversions are an *essential* part of *any* math or science related question which contains units. If you do not properly convert your units, your final answer is guaranteed to be wrong.

To highlight the importance of unit conversions, consider this massive mistake by a NASA employee in 1999:

- During a mission to Mars, the Mars Climate Orbiter spacecraft needed its path of motion to be adjusted. To do this, mission control had to remotely send directions to the spacecraft on how each of its thrusters should be positioned, and how much force each thruster should exert. Unfortunately, the employee who was responsible for calculating what the required force for each thruster must be, forgot to convert his units from imperial to metric.

This led to the spacecraft being sent 60 miles off course, costing NASA a total of $125 million dollars.

In the upcoming problem set, and from examples earlier in this lesson, it's likely that you are/will be unfamiliar with a few of the units which we have/will be been using. For this reason, I've tried my best to avoid having any unit conversions for units that you won't be familiar with.

If you're curious about some of the units, feel free to ask me. I will also have a unit conversion chart with me if you want to look at it.

Example 6:

(1) How fast is 1 meter per second in terms of kilometers per hour?

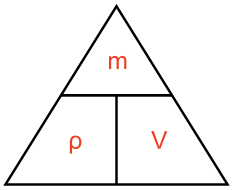
$$\frac{1\, meter}{second} \times \frac{1\, kilometer}{1000\, meters} \times \frac{60\, second}{minute} \times \frac{60\, minutes}{hour} = 3.6\, \frac{kilometers}{hour}$$

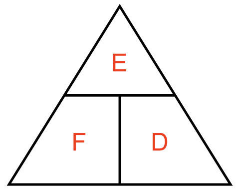
(2) How many seconds are there in 3.5 years?

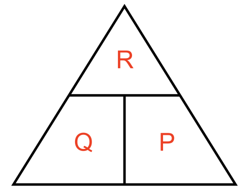
$$3.5\, years \times \frac{365\, days}{year} \times \frac{24\, hours}{day} \times \frac{60\, minutes}{hour} \times \frac{60\, seconds}{minutes} = 110\, million\, seconds$$

Problems

1. Fill in these triangles for the *Desntiy*, *Energy* and *Revenue* equations. Write all three variations of each equation as well.







$\underline{\rho = \frac{m}{V}}$ $\underline{F = \frac{E}{D}}$ $\underline{Q = \frac{R}{P}}$

$\underline{V = \frac{m}{\rho}}$ $\underline{D = \frac{E}{F}}$ $\underline{P = \frac{R}{Q}}$

$\underline{m = \rho \times V}$ $\underline{E = F \times D}$ $\underline{R = Q \times P}$

For *Density*, because the letter *D* is already used for *Distance*, a commonly used symbol is the greek letter 'ρ' (rho).

2. Water has a density of 1 kilogram per liter. (*Hint: 1 liter = 1000 milliliters*)

(a) How heavy is the water in a standard 500 milliliter water bottle?

$$m = \rho \times V = \frac{1\,kilogram}{Liter} \times 0.5\ Liters = 0.5\ kilograms$$

(b) How heavy is the water in a 2,500,000 liter olympic swimming pool?

$$m = \rho \times V = \frac{1\,kilogram}{Liter} \times 2,500,000\ Liters = 2,500,000\ kilograms$$

(c) How many 500 mL water bottles would it take to fill up the swimming pool in part b?

$$2,500,000\,Liters \div \frac{500\,milliliters}{water\,bottle} = 2,500,000\,Liters \div \frac{0.5\,Liters}{water\,bottle} = 5,000,000\,water\,bottles$$

3. A standard lightbulb consumes 60 Joules of energy every second. A standard human consumes 2,200 Calories of energy per *day*. How many days would it take a lightbulb to use the same amount of energy as a human does in a day?

$$2,200\ Calories \times \frac{4184\,Joules}{Calorie} = 9,204,800\ Joules$$

$$9,204,800\ Joules \times \frac{1\,second}{60\,Joules} \times \frac{1\,minutes}{60\,seconds} \times \frac{1\,hour}{60\,minutes} \times \frac{1\,day}{24\,hours} = 1.776\ days$$

4. A startup tech company sells virtual reality headsets for \$350 a piece. If each headset requires \$5 of wiring, two \$25 screens, \$35 of software, and \$50 of casing, how much profit will the company make after selling 1,000 units?

$$Profit = R - Exp = 1000 \times (\$350 - \$5 - (2 \times \$25) - \$35 - \$50) = 1000 \times \$210 = \$210,000$$

5. Oxygen has a density of about 1.5 grams per liter. Uranium has a density of about 20,000 grams per liter. What would have a higher volume:

(a) 1,000 grams of uranium?

$V = \frac{m}{\rho} = \frac{1{,}000\,grams}{20{,}000\,\frac{grams}{liter}} = 0.05\,Liters$

(b) 0.1 grams of oxygen?

$V = \frac{m}{\rho} = \frac{0.1\,grams}{1.5\,\frac{grams}{liter}} = 0.0\overline{6}\,Liters$

The oxygen has a higher volume!

6. The speed of light is approximately 300,000 kilometers per second.

(a) If the distance to the sun is around 150,000,000 kilometers, how many *minutes* does it take for the light from the sun to reach Earth?

$T = \frac{D}{S} = \frac{150{,}000{,}000\,kilometers}{300{,}000\,\frac{kilometers}{second}} = 500\,seconds = 8.\overline{3}\,minutes$

(b) If the radius of the Earth is 6300 kilometers, how many times can light go around the planet each second?

$C = 2\pi \times R = 2\pi \times 6300\,kilometers = 39,584\,kilometers$

$T = \frac{D}{S} = \frac{39{,}584\,kilometers}{300{,}000\,\frac{kilometers}{second}} = 0.132\,seconds$

Light goes around the Earth every 0.132 seconds. That means in 1 second, light can go around the entire planet roughly 7.5 times!

7. How far would you have to walk, while applying a 50 Newton force, to burn 100 Joules of energy? How about 100 Calories?

$D = \frac{E}{F} = \frac{100\,Joules}{50\,Newtons} = 2\,meters$

For 100 Calories, you will have to walk 4,184 times as far, because you will use 4,184 times the energy! $D = 2\,meters \times 4,184 = 8,368\,meters$

8. A certain company made \$12,500,000 of revenue in 2016 from one of their products. If they made \$4,000,000 in profit, and every unit they sold had \$125 of expenses, how many units did they sell in 2016?

$Expenses = Revenue - Profit = \$12,500,000 - \$4,000,000 = \$7,500,000$

$Total\ Number\ Of\ Units = \frac{Total\ Expenses}{Expense\ Of\ One\ Unit} = \frac{\$7{,}500{,}000}{\$125} = 68,000\,units$

*9. Arnold found a mysterious object when digging around near the center of the Earth. He notices that when he pushes on the object with a force of 100 Newtons, it begins to accelerate at a speed of 2 meters per second squared.

(*Hint: Use one of the equations in the second part of Example 3*)

(a) How heavy is the object? (Your answer will be in kilograms)

$mass = \frac{Force}{acceleration} = \frac{100\,Newtons}{2\,\frac{meters}{second^2}} = 50\,kilograms$

(b) If the object has a volume of 2.5 liters, what is its density?

$\rho = \frac{mass}{volume} = \frac{50\,kilograms}{2.5\,Liter} = \frac{20\,kilograms}{Liter}$

(c) What material is this object likely made from?

$\rho = \frac{20\,kilograms}{Liter} = \frac{20{,}000\,grams}{Liter}$

This is the same density that Uranium had in question 5! We can assume that because this unknown object has the same density as Uranium that it actually is Uranium!

**10. Power is a measurement of how much energy something uses in a given amount of time.

(a) Based off of this, what do you think an equation for power would be?

Similarly to the density equation where we knew density was an amount of mass per volume, here we are given power is an amount of energy per time.

Because we have an amount of something per an amount of something else, we can agin write out equation as a ratio of the two things.

$$Power = \frac{Energy}{Time}$$

(b) The unit used to measure power is known as a 'watt'. How many watts does the Ferrari 488 GTB use if it can travel at a top speed of 90 meters per second while exerting a force of 5000 Newtons?

$Power = \frac{Energy}{Time} = \frac{Force \times Distance}{\frac{Distance}{Speed}} = Force \times Speed$

Using this new equation for power, we get:

$Power = Force \times Speed = 5000\,Newtons \times \frac{90\,meters}{second} = 450,000\,Watts$

(c) What is the horsepower rating of the Ferrari 488 GTB?

(*Hint: 1 Horsepower = 750 Watts*)

$450,000\,Watts \div \frac{750\,Watts}{Horsepower} = 600\,Horsepower$