Math Circles
25 October 2017
Logic and Problem Solving, Part 3
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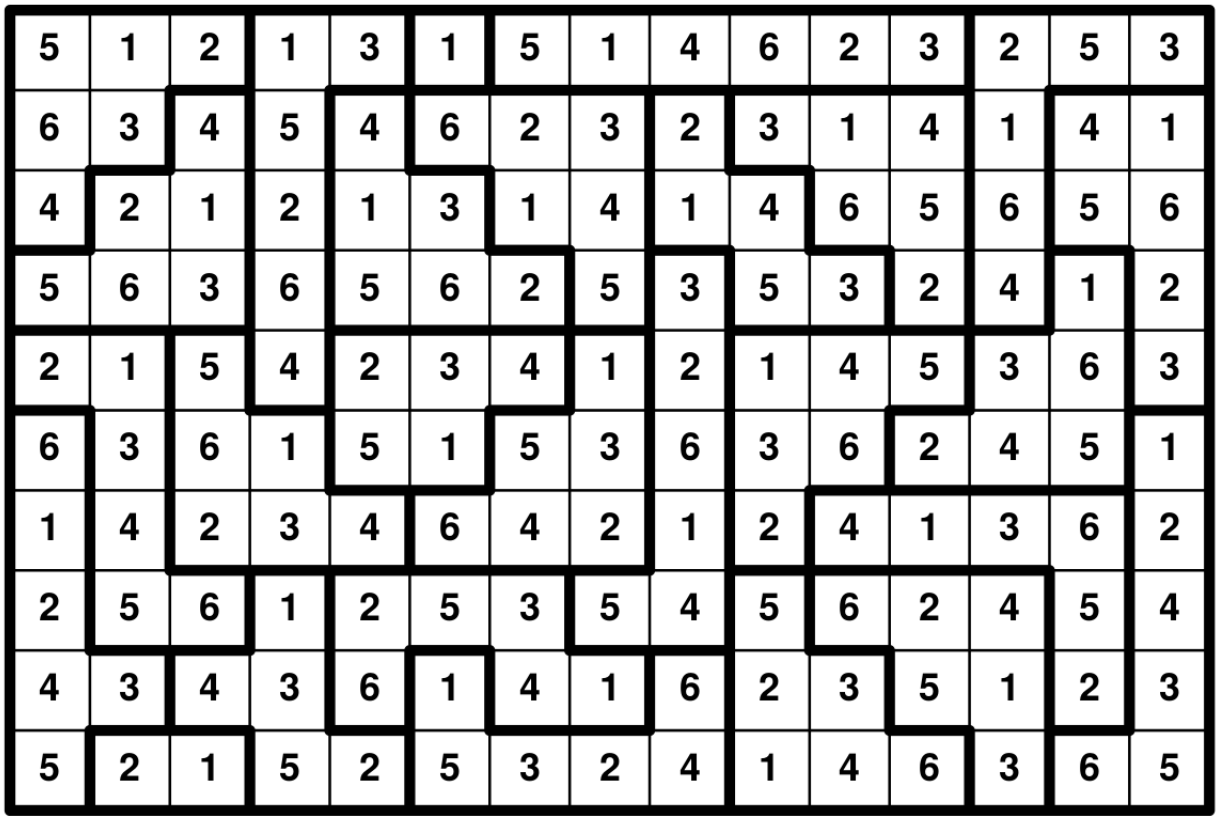
EXERCISES

1. The puzzle below is a *suguru*. As in Kenken, the heavy lines indicate "cages", and these bound a region from one to six squares in size. Fill each square with unique digits, counting up from 1. For example, if the cage has 3 squares in it, you must put the numbers 1, 2 and 3 in it. Squares which touch, including those squares which touch diagonally, may never contain the same number.

5				3				4	6			2	5	
6						2				1				
4					3		4					6	5	
						2							1	
2		5							1					3
6					1			6		6		4		1
	4		3									3		
2		6			5			4					5	4
			3		1	4		6			5			3
	2				5		2		1	4		3	6	

Solution:



2. Consider placing a queen chess piece anywhere on the 8×8 board. Once placed, the queen can move left, down or diagonally down-left any number of squares. (These are half of the normal movements a queen can make.)

   Two players, A and B, alternate moves. Player A goes first.

   The goal is to move the queen into the bottom-left cell, which is marked with an X. The first player to do so wins.

X							

(a) Fill in the remaining 63 squares with either A or B to indicate who is guaranteed to win if the queen is placed in that position.

Solution:

A	A	A	A	**B**	A	A	A
A	A	A	A	A	A	A	A
A	A	A	**B**	A	A	A	A
A	A	A	A	A	A	A	**B**
A	A	A	A	A	**B**	A	A
A	**B**	A	A	A	A	A	A
A	A	**B**	A	A	A	A	A
X	A	A	A	A	A	A	A

(b) Model the problem as Nim game with two piles. Clearly state what the initial piles are, what the rules of removal are and what the winning condition is. You do not need to prove how the Nim version is won.

Solution:

Initially, start with two piles of N sticks each.

Players may take any number of sticks from either pile (this is a horizontal or vertical move), or the same number of sticks from both piles (this is a diagonal move).

The winner is the person who takes the last stick.

3. Consider a 3×3 Game of Life board. Recall that in the Game of Life, if a cell is populated (alive):

- Each cell with 0 or 1 neighbour dies (due to solitude)
- Each cell with 4 or more neighbours dies (due to overpopulation)
- Each cell with 2 or 3 neighbours survives.

If a cell is unoccupied, each cell with exactly 3 neighbours becomes populated.

Recall also that an interior cell will have 8 neighbours, but corner cells will only have 3 neighbours, and edge cells (not in the corner) will have 5 neighbours.

(a) How many different configurations are there? Explain.

(b) What happens to an initially full grid (i.e., where all cells are populated initially)? Explain your reasoning.

(c) How many different configurations are "steady state"? That is, how many configurations do not change after each generation? Explain. There are between 10 and 15.

Solution:

(a) $2^{16} = 65536$ different configurations. There are $4 \cdot 4 = 16$ cells, and each can be alive or dead, thus in 2 configurations, and each cell is independent of the other cells.

(b) Consider alive cells being marked with an X and dead cells being empty.

X	X	X
X	X	X
X	X	X

Notice that the only cells which remain alive are the 4 corner cells: every other cell has either 5 or 8 neighbours, all of which were alive, and thus they die. We then get

X		X
X		X

None of the alive cells have any neighbours, and none of the dead cells has 3 neighbours, and thus, all cells die to become an empty grid:

(c) There are 12 steady states.

From part (b), we have the empty grid being steady state.

There are four steady states with a "block" (consider shifting it right, down and right/down):

X	X	
X	X	

There is one steady state as a "diamond":

	X	
X		X
	X	

There are four steady states of the "diamond plus one corner":

	X	
X		X
X	X	

There are two steady states of the "diamond plus opposite corners":

	X	X
X		X
X	X	

4. Write a computer program to find all steady states in the 4x4 (or even larger) Game of Life.

   Solution:
   No solution provided: try to write one up in your favourite programming language. There are 158 steady states in the 4x4 case and 725 steady states for the 5x5 case.