Intermediate Math Circles
Winter 2018
Even More Fun With Inequalities

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What Happened Last Week?

- We went over graphing inequalities on a number line
- I took a mulligan when it came to explaining absolute values
- Somewhere in there I made I struggled with the difference between AND and OR
- We saw a Beaver Computing Challenge question relating to binary search
- We quickly went over graphing inequalities with two variables
- I tried to get you to play battleship with 10 minutes left

To summarize, I talked way too much and didn't give you enough time to work on problems.

Last Absolute Value Question

Solve $|x-3|+|x+4|>9$ graphically.

Words Hard

OR as a conjunction (joining word)

Ex. to be or not to be

OR as a mathematical operator.

Let A and B be statements that are either true or false.

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

Last Absolute Value Question

Using the approach I showed you last week how many cases would you need to consider to solve $|x-3|+|x+4|>9$ algebraically?

Basic Properties

There are eight basic properties for $\leq$ and their names are in brackets on the right. For all the properties x,y,z, and r are real numbers.

(1) $x \leq x$ (reflective)

(2) If $x \leq y$ and $y \leq x$, then $x = y$ (antisymmetric)

(3) If $x \leq y$ and $y \leq z$, then $x \leq z$ (transitive)

(4) One of the following three holds: $x < y$, $y < x$, or $x = y$ (trochotomy)

(5) If $x \leq y$, then $x + r \leq y + r$

(6) If $x \leq y$ and $0 \leq r$, then $rx \leq ry$

(7) If $x \leq y$ and $r \leq 0$, then $ry \leq rx$

(8) $0 \leq x^2$

Linear Inequalities With Two-Variables

Solving linear inequalities is much like solving linear equalities but with one exception **exception**. The exception being, if we multiply or divide both the left and the right hand sides by a negative number we **flip** the inequality.

Graphing linear inequalities with two variables is just like graphing linear equalities with two variables. The difference being there is an extra step required to determine the region which satisfies the inequality.

Linear Inequalities With Two-Variables

Procedure:

1. Change your inequality to equality
2. Graph that equation
3. Determine the region that satisfies the inequality using one of the two methods below.

Mike's Method

Pick a point in one of the two regions divided by the line. If the point satisfied the inequality, that region represents the set of points which satisfy the inequality. If the point doesn't, the set of points is the other region.

Radford's Method

Set one of the variables to a specific value and then evaluate the single variable inequality for the other variable.

Practice

Graph the region that satisfies all three of these inequalities

$$3x - y \leq 12$$
$$x + y < 5$$
$$x - 2y > 4$$

I.e. graph the region that satisfies $3x - y \leq 12 \cap x + y < 5 \cap x - 2y > 4$

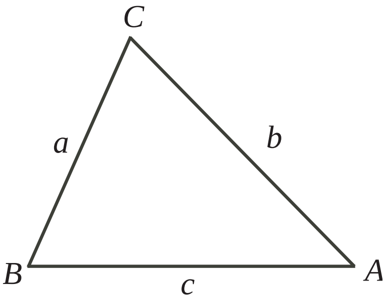
What you will need

- Overview and instructions page
- Graph paper with lattice points
- Tracking page
- Ruler

WARNING

Remember to read the instructions carefully and be aware of nuances.

Angle-Side Inequality



If $\angle A < \angle B$, then $a < b$.

If $a < b$, then $\angle A < \angle B$.

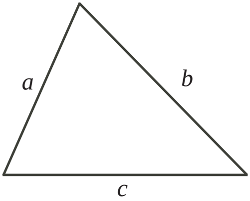
Toothpick Experiment

The last topic we are going to cover with inequalities is the *Triangle Inequality*, but before we formally state it we need to play around with the idea.

For the toothpick experiment you are going to need

- Toothpick experiment handout
- 12 toothpicks

Triangle Inequality



If a, b and c are the side lengths of a triangle, the Triangle Inequality tells us that

$$b + c > a$$
$$a + c > b$$
$$a + b > c$$

What happens if we use $\geq$ instead of $>$?
What assumptions are we making about the side lengths?