



# Intermediate Math Circles

## February 21, 2018

### Contest Prep III

#### Equations and Algebra

First: systems of linear equations

Second: systems of non-linear equations

#### Degree of a Term

Every term has a degree. The degree of the term is the sum of all of the exponents in the term. What is the degree of each of the following terms?

1.  $x$
2.  $17x$
3.  $x^3y^2$
4.  $xyz$

#### Linear Equation

An equation is linear if all exponents on the variable is a non-negative integer and the highest degree of all of the terms is 1. Otherwise it is non-linear.

Examples:

- $x - 2y - z = 1$
- $x^2 + y^2 = 4$
- $\sqrt{x} - \sqrt{y} = 1$
- $x + xy = 3$

#### Systems of Linear Equations:

Two equations, two unknowns:

**Example 1:** Solve the following system of equations for  $x$  and  $y$ :

$$2x + y = 5$$

$$x - 3y = 6$$

We can use **substitution** or **elimination**:



**Substitution:**

Use one equation to isolate for one of the variables, then substitute into the other equation.

$$2x + y = 5 \quad (1)$$

$$x - 3y = 6 \quad (2)$$

Choose an equation and solve for  $y$  in terms of  $x$  or  $x$  in terms of  $y$ :



**Elimination:**

Solve

$$2x + y = 5 \quad (1)$$

$$x - 3y = 6 \quad (2)$$

When performing operations on equations, we can only perform operations that do not change the solution set. The allowed operations are:

1. multiply any equation by any non-zero constant
2. add any two equations
3. subtract any two equations



The techniques used to solve systems of two linear equations with two unknowns can be used to solve systems of linear equations with more equations and more unknowns:

**Example 2:**

My dog Bernie would not stay still while trying to weigh him at the pet clinic. To help find his weight, I held Bernie and stood on the scale while the nurse read off 100 kg. Then the nurse held Bernie and stood on the scale while I read off 82 kg. Finally, I held the nurse and stood on the scale while Bernie read off 150 kg (yes, Bernie can read!). Determine my weight, Bernie's weight and the nurse's weight.

**Solution:**

## Systems of Non-linear equations:

Use some combination of

- knowledge of how to solve systems of linear equations (substitution, elimination)
- factoring and regrouping
- creativity (add/subtract equations, multiply equations, etc.)

## Some useful things to know about multiplying monomials and binomials:

$$2(x + y) =$$

$$a(x + y) =$$

What if we replace  $a$  with  $2x + y$ ?

$$(2x + y)(x + y) = ($$

In general,

$$(a + b)(x + y) =$$

A special case:

$$(x - y)(x + y) =$$

**“difference of squares” formula**

## Some useful things to know about solving equations:

If  $A \times B = 0$ , then either  $A = 0$  or  $B = 0$ .

If  $ax + b = 0$  and  $a \neq 0$ , then  $x = -\frac{b}{a}$ .

If  $ax^2 + bx + c = 0$  and  $a \neq 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

the **“quadratic formula”**



**Example 3:** Solve the following system of equations

$$\begin{aligned}\sqrt{x} + \sqrt{y} &= 36 \\ x - y &= 144\end{aligned}$$



**Example 4:** Determine all  $(x, y)$  pairs where  $x$  and  $y$  are positive integers and  $x + xy = 391$ .