Intermediate Math Circles
Contest Preparation I

Part A: Each question is worth 5 credits.

1. $1995 - 995 + 95$ equals

   (A) 905 **(B)** 1905 **(C)** 195 **(D)** 1095 **(E)** 1995

2. The value of $\frac{5 \times 4 \times 3}{5 + 4 + 3}$ is

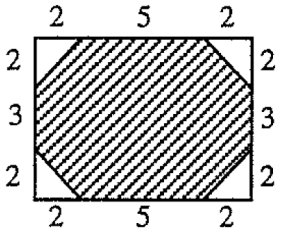
   (A) 5 **(B)** 1 **(C)** $\frac{1}{3}$ **(D)** 19 **(E)** 3

3. If $p = 3$, the value of $(p-3)(p-2)(p-1)$ is

   (A) 0 **(B)** 2 **(C)** -6 **(D)** 3 **(E)** 21

4. In the 9 by 7 rectangle shown, what is the area of the shaded region?

   (A) 15 **(B)** 55 **(C)** 16
   (D) 59 **(E)** 47



5. The expression $2^5 - 2^4$ has a value equal to

   (A) 0 **(B)** 2 **(C)** 2^{-1} **(D)** 2^3 **(E)** 2^4

6. A rectangular picture, 1.2 m wide, is centred on a wall that is 5 m wide. What is the distance, in metres, from an edge of the wall to the nearer edge of the picture?

   (A) 3.1 **(B)** 1.3 **(C)** 1.9 **(D)** 0.6 **(E)** 3.8

7. Two numbers are selected from $-9, -7, -5, 2, 4$, and 6. If they are multiplied, the least possible value for this product is

   (A) -63 **(B)** -54 **(C)** -18 **(D)** -10 **(E)** 8

8. The largest prime number that divides 1995 exactly is

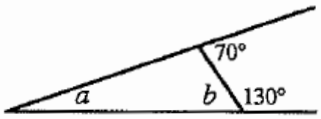
   (A) 399 **(B)** 19 **(C)** 5 **(D)** 7 **(E)** 133

9. A bag contains 5 yellow marbles, 10 red marbles, and 15 white marbles. If one marble is selected at random, the probability that it is yellow is

   (A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{30}$

10. In the diagram, the value of $a+b$, in degrees, is

(A) 70 (B) 200 (C) 130
(D) 100 (E) 160



Part B: Each question is worth 6 credits.

11. If $2^x+3^y=43$, where x and y are integers, then the value of $x+y$ is

(A) 9 (B) 8 (C) 7 (D) 6 (E) 5

12. A piggy bank holds \$5.64 in pennies, nickels, dimes, quarters, and dollar coins. If there is an equal number of coins of each kind, then the total number of coins is

(A) 5 (B) 10 (C) 15 (D) 16 (E) 20

13. The value of $\left(12.5\times10^{-3}\right)\times\left(8\times10^{111}\right)$ is equal to the value of

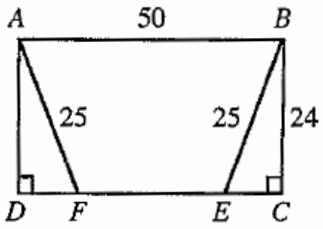
(A) 10^{110} (B) 1^{110} (C) 10^{39} (D) 100×10^{-333} (E) 1000^{108}

14. A team of 18 Canadians will play a series of games. If three team members must sit out each game, the smallest number of games that will allow each one to play an equal number of times is

(A) 30 (B) 15 (C) 18 (D) 5 (E) 6

15. $ABCD$ is a rectangle with F and E on DC as shown. The perimeter of quadrilateral $ABEF$ is

(A) 114 (B) 148 (C) 100
(D) 136 (E) 134



16. The numbers 1, 2, 3, 4, and 5 are to be placed in the grid, in such a way that each can occur only once in any row, column, or diagonal. The value of $P+Q$ is

(A) 6 (B) 10 (C) 8
(D) 9 (E) 7

1				
	2		5	
		3	Q	1
				3
	3		1	P

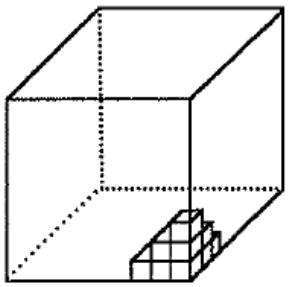
17. The number of ways that 20 can be expressed as the sum of exactly three prime numbers is

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

18. If $\frac{2}{5}x=16-\frac{2}{5}x$, then $\frac{5x}{2x+10}$ equals

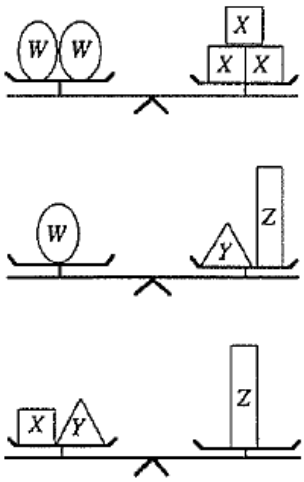
(A) 2 (B) $\frac{1}{4}$ (C) $\frac{25}{11}$ (D) $\frac{10}{9}$ (E) 0

19. A number of small 1 cm × 1 cm × 1 cm cubes are glued together to form a solid 10 cm × 10 cm × 10 cm cube. The outside of the large cube is then painted. The number of these small cubes with exactly two faces painted is



(A) 60 (B) 96 (C) 104
(D) 64 (E) 112

20. The scales in the diagram are balanced. The number of Y's that would be required to balance one Z is

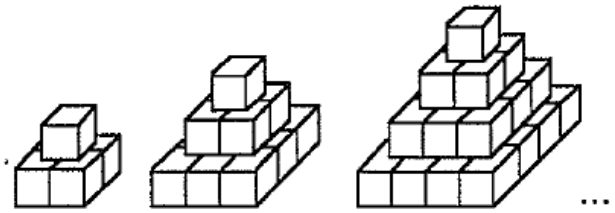


(A) 4 (B) 6 (C) 3
(D) 5 (E) 2

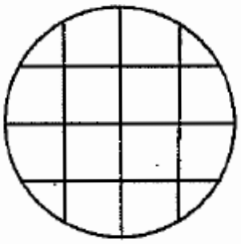
Part C: Each question is worth 8 credits.

21. The Ancients built square-based structures similar to those shown in the diagram. They began with 1000 identical cubes and wished to build as many structures as possible. The first structure contained two layers. Beside it was constructed a second structure with three layers. This process was continued as shown until the number of cubes left was not sufficient to build the next structure. How many cubes were left?

(A) 175 (B) 176 (C) 209 (D) 181 (E) 182



22. Two perpendicular diameters are drawn in a circle of radius 2. All possible chords parallel to and at a distance of 1 unit from these diameters are drawn. The total length of the six chords in the diagram (including the two diameters) is



(A) $8(\sqrt{5}+1)$ (B) $4(\sqrt{3}+2)$ (C) 22
(D) $8(\sqrt{3}+1)$ (E) $4(\sqrt{5}+2)$

23. If $21xy^2$ and $15xy$ are perfect squares, where x and y are integers greater than zero, the least value of $x+y$ is

(A) 36 (B) 56 (C) 26 (D) 16 (E) 46

24. The sum of 28 consecutive odd, positive integers is a perfect cube. If p and q are the least and greatest of these integers, the average of the least possible value for p and the least possible value for q is between

(A) 80 and 90 (B) 90 and 100 (C) 100 and 110 (D) 110 and 120 (E) 120 and 130

25. In a trapezoid the parallel sides are in the ratio $k:1$, where $k>1$. The line segment joining the midpoints of the non-parallel sides divides the trapezoid into two regions whose areas are in the ratio 5:2. What is the value of k?

(A) 12 (B) $\frac{5}{2}$ (C) 13 (D) $\frac{25}{4}$ (E) 7

Answers to Contest Preparation 1:

1. D	2. A	3. A	4. B	5. E	6. C
7. B	8. B	9. A	10. A	11. C	12. E
13. A	14. E	15. D	16. E	17. C	18. A
19. B	20. D	21. B	22. D	23. B	24. B
25. C					