

Problem Set 1

Intermediate Math Circles Winter 2018
Fun With Inequalities

Linear Inequalities- Single Variable

Solve each of the following.

$$1. \ x + 5 < \frac{7}{2}x < \frac{7}{2} - 5$$
$$x < \frac{7}{2} - \frac{10}{2}$$
$$x < \frac{-3}{2}$$

$$2. \ 3 - \frac{x}{2} \geq -8$$

$$3. \ -1 - 3x \leq 4x + 10$$

$$4. \ 2x + 5 > 4x - 7$$

$$5. \ -\frac{2}{3}x + \frac{3}{7} \leq 5 - \frac{x}{2}$$

Absolute Values

Solve each of the following algebraically. Check your answer graphically.

$$1. \ |x + 6| = 5$$

$$2. \ |x - 4| \geq 1$$

$$3. \ |4 - x| \geq 1$$

$$4. \ |2x + 1| < 7$$

$$5. \ |x - 2| + |x + 5| = 8$$

$$6. \ |x| + |2 - x| \leq 12$$

Properties

1. Which of the eight properties of \leq also hold for $<$?
2. Use whichever of the properties (1) to (8) that you need to prove the following
 - (a) If $a \leq b$ and $c \leq d$, then $a + c \leq b + d$.
 - (b) If $0 \leq a \leq b$ and $0 \leq c \leq d$, then $0 \leq ac \leq bd$.
3.
 - (a) If $a \leq b$ and $c \leq d$, is it true that $ac \leq bd$?
 - (b) If $a \leq b$, is it true that $\frac{1}{b} \leq \frac{1}{a}$?
4. Show that if $a < b$, then $a < \frac{1}{2}(a + b) < b$.
5. Show that the sum of a positive number and its reciprocal is at least 2.
In other words show that

$$a + \frac{1}{a} \geq 2$$