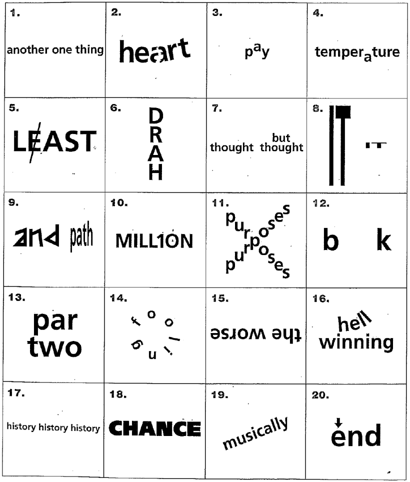
Wacky Wordies

The object of this activity is to discern a familiar phrase, saying, cliche, or name from each arrangement of letters. For example, in box #1 the phrase is “one thing leads to another”.



Intermediate Math Circles
Winter 2018
Fun With Inequalities

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Erik's Approximation

My friend Erik once told me that an easy way to convert from temperature in Celsius to Fahrenheit is take the Celsius temperature double it and add 30°.

The actual conversion from Celsius to Fahrenheit is done using the following linear equation

$$f = \frac{9}{5}c + 32^\circ$$

where f represents the Fahrenheit temperature and c represents the Celsius temperature.

a. For $30^\circ C$, what is the difference between Erik's and the actual Fahrenheit temperature?
b. For what Celsius temperature does Erik's conversion give the correct Fahrenheit temperature?
c. When does Erik's conversion give a greater Fahrenheit temperature than the actual?

What Should Be Review

At this point in your mathematical careers I am sure you have seen the symbols

$$>, <, \geq, \text{ and } \leq$$

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Which of the following are true and which are false?

1. $27 < 72$
2. $-27 < 72$
3. $-27 \leq -72$
4. $27 \leq 27$

What Does $\leq$ Mean?

Less than or equal to

Given two real numbers, a and b, we know that $a \leq b$ if a is equal to b or lies to the left of b on the real number line.

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Instead we should have $-72 \leq -27$

Basic Properties

There are eight basic properties for $\leq$ and their names are in brackets on the right. For all the properties x,y,z, and r are real numbers.

(1) $x \leq x$ (reflective)

(2) If $x \leq y$ and $y \leq x$, then $x = y$ (antisymmetric)

(3) If $x \leq y$ and $y \leq z$, then $x \leq z$ (transitive)

(4) One of the following three holds: (trochotomy)
$x < y$, $y < x$, or $x = y$

(5) If $x \leq y$, then $x + r \leq y + r$

(6) If $x \leq y$ and $0 \leq r$, then $rx \leq ry$

(7) If $x \leq y$ and $r \leq 0$, then $ry \leq rx$

(8) $0 \leq x^2$

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Consider our earlier example with Erik's approximation.

Part b)	Part c)

Solving Linear Inequalities (Single Variable)

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$2c + 30 = \frac{9}{5}c + 32$	$2c + 30 > \frac{9}{5}c + 32$

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What's the exception?

The Exception

Remember the Property (7)

$$\text{If } x \leq y \text{ and } r \leq 0\text{, then } ry \leq rx$$

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$$\text{If } x \leq y \text{ and } r \leq 0 \text{, then } ry \leq rx$$

What does this mean to multiply an inequality by a negative number?

It means that if you are reflecting your values about zero on the number line and for the inequality to still hold the values need to be flip.

Geogebra Experiment #1

To get a better idea what this reflection looks like we are going to open the Geoebra file *Inequalities- Property 7*.

The Exception

For solving inequalities, Property (7) requires us to **flip** the inequality if we multiply or divide both the left and the right hand sides by a negative number.

Example

Solve $3x < 16x - 52$

Is there a way to do this without multiplying or dividing by a negative?

Proving Property (7)

We have given visual justification that Property (7) holds. However it wouldn't hurt to prove it holds algebraically.

Proof.

□

The Red Shirt or Blue Shirt

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What does this look like in "math speak"?

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$$|b| = \begin{cases} b & \text{if } b \geq 0 \\ -b & \text{if } b < 0 \end{cases}$$

Another cool way of expressing *absolute value* is as follows

$$|b| = \sqrt{b^2}$$

Solving Absolute Value Inequalities (Single Variable)

Examples

(a) $|0|$ (b) $|7|$ (c) $|-3|$

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Geogebra Experiment # 2

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Another way to think about *absolute value* is as the distance from a *special point*. Sometimes that *special point* is zero, sometimes it is non-zero, and sometimes there are multiple *special point*. To find these *special point* we need to set what's contain in each absolute value to zero and solve.

Solving Absolute Value Inequalities (Single Variable)

Example

Solve $|x-2| > 4$

How would you do this algebraically?

Solving Absolute Value Inequalities (Single Variable)

Example Continued

Solve $|x - 2| > 4$

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Let's check this graphically.
Let's start by solving $|x-2| = 0$.
What we are trying to do is find a *special point*.

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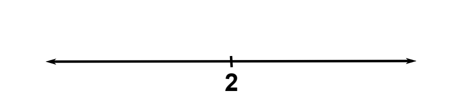

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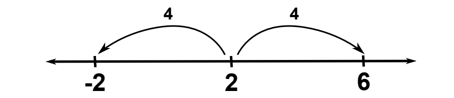

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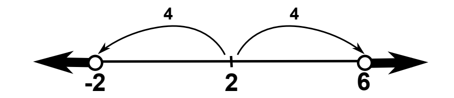

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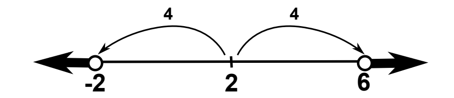

Therefore we can see that the inequality holds for $x < -2$ and $x > 6$.

Solving Absolute Value Inequalities (Single Variable)

Challenge

Solve $|x+1| + |x-2| > 4$ geometrically and algebraically.