Intermediate Math Circles
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More Fun With Inequalities

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Inequalities On a Number Line

What I assumed you had seen before... my bad

How do you express the following inequalities on a number line?

a. $x \geq 5$
b. $x < -1$
c. $-2 < x < 5$
d. $5 \leq x$ and $x \leq -11$

What we are doing is graphing in **one dimension**.

Solving Absolute Value Inequalities (Single Variable)

The |ABS| on the blue shirt is referring to *absolute value*. So what is *absolute value*?

Definition

The *absolute value* $|b|$ of a real number b is defined to be b if b is positive or zero, and to be $-b$ if b is negative.

What does this look like in "math speak"?

$$|b| = \begin{cases} b & \text{if } b \geq 0 \\ -b & \text{if } b < 0 \end{cases}$$

Another cool way of expressing *absolute value* is as follows

$$|b| = \sqrt{b^2}$$

Solving Absolute Value Inequalities (Single Variable)

Another way to think about *absolute value* is the distance from a *special point*. Sometimes that *special point* is zero, sometimes it is non-zero, and sometimes there are multiple *special point*. To find these *special point* we need to set what's contain in each absolute value to zero and solve.

Geogebra Experiment # 2

To get a better idea what this looks like we are going to open the Geoebra file *Inequalities- Absolute*.

Solving Absolute Value Inequalities (Single Variable)

Examples

Solve each of the following equations and inequalities.

1. $|x| = 12$
2. $|x| \geq 5$
3. $|x + 6| = 5$
4. $|x - 4| \geq 1$
5. $|x - 3| + |x + 6| < 13$

Solving Rational Inequalities (Single Variable)

Before we deal with inequalities let's work with equalities first.

Example

Solve each of the following equations.

a. $\dfrac{4}{x} = \dfrac{5}{6}$
b. $\dfrac{-2}{x+4} = \dfrac{3}{x-1}$

The most important thing to remember when solving rational equalities is

$$\boxed{\text{DON'T DIVIDE BY ZERO!}}$$

Only Chuck Norris can divide by zero.

Solving Rational Inequalities (Single Variable)

Solving rational inequalities is what you would expect. It is similar to solving rational equalities, expect for that pesky property 7.

Just like when solving rational equalities you need to be aware when the denominator can be zero and exclude those values from your answer.

Example

Solve the inequality $\frac{4}{x} < -\frac{5}{6}$ algebraically.

Where is the error?

Example

For our example, can you spot the error in the following solution?

$$\frac{4}{x} < -\frac{5}{6}; \qquad x \neq 0$$

$$4(6) < -5(x)$$

$$24 < -5x$$

$$\frac{24}{-5} > \frac{-5x}{-5}$$

$$-\frac{24}{5} > x$$

$$x < -\frac{24}{5}$$

$$x < -4.8$$

Solving Rational Inequalities (Single Variable)

Challenge

Solve the inequality $\dfrac{-2}{x+4} \geq \dfrac{3}{x-1}$ algebraically.

Find the Thief

The famous Blue Diamond was stolen from a museum. A thief swapped it for a cheap green imitation diamond.

The day it was stolen, 2000 people entered the diamond room one by one. Inspector Bebro must find the thief by interviewing some of these people. He has a list of all 2000 people in the order they entered the room. The only question he can ask a person is:

Was the diamond green or blue when you saw it?

The thief will lie and say green but everyone else will tell the truth.

Find the Thief

Inspector Bebro is very clever and will use a strategy where the number of people interviewed is as small as possible. Which of the following statements can he say truthfully?

(A) I can guarantee that I will find the thief by interviewing at most 20 people.

(B) Interviewing 20 people will not be enough (unless I am lucky) but I can certainly find the thief by interviewing fewer than 200.

(C) This is going to be a difficult job: I will need to interview at least 200 people, but possibly as many as 1999 people.

(D) I cannot promise anything. If I am very unlucky, I might need to interview every single visitor.

Solving Linear Inequalities (Two Variables)

Solving linear inequalities is much like solving linear equalities with the same **exception**.

That is, if we multiply or divide both the left and the right hand sides by a negative number we **flip** the inequality.

Practice

Graph the line $2x + 3y = 6$.

Solving Linear Inequalities (Two Variables)

Example

Now given the line $2x + 3y = 6$, what does the region that satisfies the inequality $2x + 3y < 6$ look like with respect to our original line?

How do you know where this region (or set of points) is with respect to the line?

Solving Linear Inequalities (Two Variables)

Mike's Method

Pick a point in one of the two regions divided by the line. If the point satisfied the inequality, that region represents the set of points which satisfy the inequality. If the point doesn't, the set of points is the other region.

Radford's Method

Set one of the variables to a specific value and then evaluate the single variable inequality for the other variable.

Solving Linear Inequalities (Two Variables)

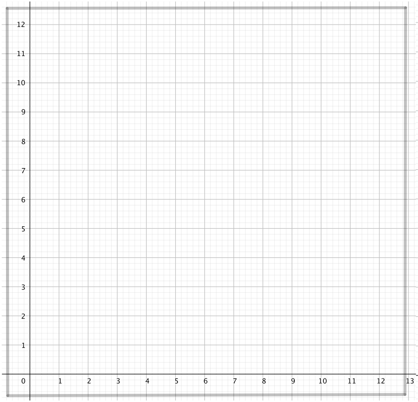
Practice

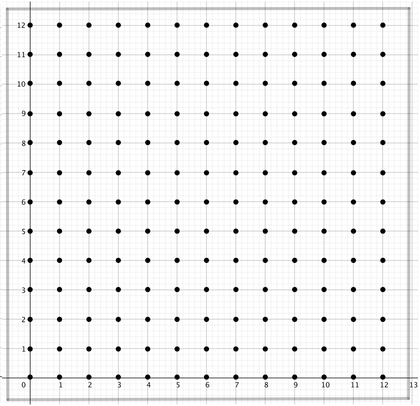
Graph the following inequalities.

1. $2y - x \geq 2$
2. $2x + 3y < 6 \quad \cap \quad 2y - x \geq 2$

Inequality Battleship

We are going to play a game of battleship. The region in the x- and y-axes we are going to be working with is all the lattice points where $0 \leq x \leq 12$ and $0 \leq y \leq 12$. A point (x, y) with both x and y coordinates integral is called a lattice point.

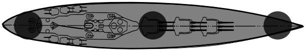




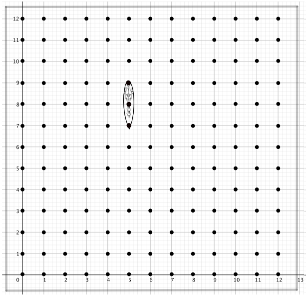
Inequality Battleship

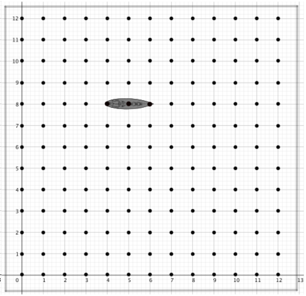
There are going to be a couple similarities and a couple variations with our battleship game versus the classic version.

- There will only be one ship and it is three lattice points long.



- Like the classic version the ship can only be placed horizontally or vertically.





Inequality Battleship

When it comes to firing torpedos in this version of battleship we are going to be using mostly inequalities like the ones we just saw.

However, there are a few restrictions

- You do have to make guesses about what lattice points the ship is on, but you only get four lattice point guesses.
- You can only start guessing lattice points after you have made three inequalities guesses.
- The lines for your inequality guesses cannot have the same slope

Inequality Battleship

Supplies: Each player will need one of the following to play a game

- Ruler
- Battleship board pages
- Torpedo tracking pages

Inequality Battleship

Instructions: As you might have guessed this is a two player game

- Find an opponent
- Arrange it so you are facing back-to-back
- Each player places their battleship on the provided board pages
- The older player is the first player and the younger the second
- First player places their guess on their tracking sheet and passes it to the second player. Second player checks to see if <u>any of their battleship's lattice points satisfy the guess</u>.
- Return the sheet back to the first player with yes or no in the appropriate column
- Second player now does the same with their tracking sheet.
- This repeats until one player finds all the lattice points of their opponents battleship