

Math Circles - Problem Set 2

Linear Diophantine Equations cont.

Zack Cramer - zcramer@uwaterloo.ca

March 7, 2018

1. Find the full list of solutions to the following LDEs:
 - (a) $94x + 44y = 12$
 - (b) $1002x + 954y = 42$
2. Becky wants exactly 600 of her daily calories to come from green eggs and ham. Each slice of ham has 102 calories, and each egg has 18 calories. Find all combinations of green eggs and ham will total 600 calories?
3. A strange virus that turns people into goombas has infected the entire city of Toronto. Luckily, one of the city's top scientists was able to send you a distress email before she was infected. The email contains the following information on creating a cure:
 - Each person can be cured by consuming exactly 1812mg of chemical Z.
 - Chemical Z can only be found in pickles and marshmallows.
 - Each pickle contains 312mg of chemical Z.
 - Each marshmallow contains 252mg of chemical Z.

How many pickles and marshmallows should each person be given in order to restore the city to normal?

Challenge Problems

4. Let a, b , and c be positive integers. Show that the number of non-negative integer solutions to the LDE

$$ax + by = c$$

cannot exceed c/a or c/b .

Problems from Lesson 1

- Use the Euclidean algorithm to compute the following GCDs:
 - $\gcd(204, 99)$
 - $\gcd(1\,053, 993)$
 - $\gcd(7\,404, 7\,032)$
- Find a solution for each of the following LDEs, or explain why one does not exist.
 - $204x + 99y = 3$
 - $1\,053x + 993y = 7$
 - $7\,404x + 7\,032y = 36$
- Can 10 000 be expressed as a sum of two integers, one of which is divisible by 126 and the other divisible by 81? If so, find examples of such integers. If not, explain why.
- Can 10 000 be expressed as a sum of two integers, one of which is divisible by 614 and the other divisible by 72? If so, find examples of such integers. If not, explain why.
- Find an integer n , which, when divided by 78 leaves a remainder of 37; and when divided by 29 leaves a remainder of 17.
- Use the Euclidean algorithm to find a solution to $25x + 10y = 215$, the LDE from example (III).
 - Does your answer in part (a) make sense in the context of the problem? If not, how can we find a solution that does make sense?

Challenge Problems

- Use the division algorithm to show that $\gcd(k + 1, k) = 1$ for any integer k .
 - Use the division algorithm twice (i.e., the Euclidean algorithm) to show that
$$\gcd(7k + 6, 6k + 1) = 1$$
for any integer $k \geq 1$.
- Let a and b be integers. For which integers c does $ax + by = c$ have a solution?
 - Let a, b , and c be integers. For which integers d does $ax + by + cz = d$ have a solution?
 - Find a solution to the 3-variable LDE $18x + 14y + 63z = 5$.