



Grades 7 & 8, Math Circles

13/14/15 February, 2018

Circuits

Introduction

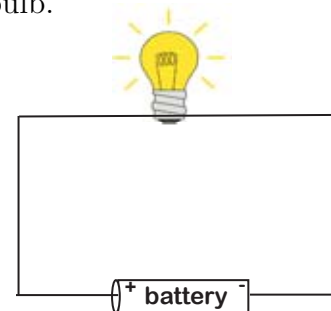
Electricity is an intricate part of our everyday lives and comes in two main forms, static electricity and current electricity. _____ is when electricity gathers at one spot, like when you rub a balloon against your head! _____ is when electricity moves from one spot to another such as the electrons moving through an extension cord.

Today we are going to look at current electricity and some of the mathematics involved in electric circuits. Let's begin with some terminology.

Terminology

A _____ is a closed path that allows for an electrical current to flow through. An example would be the wires connecting a power source to a light bulb.

_____ is a steady flow of electrons that moves throughout the circuit. It is measured in amperes (or amps) and represented by the symbol "I".



_____ is the electrical force that makes the electric current move through the wire. We represent it using a "V" and it is measured in _____.

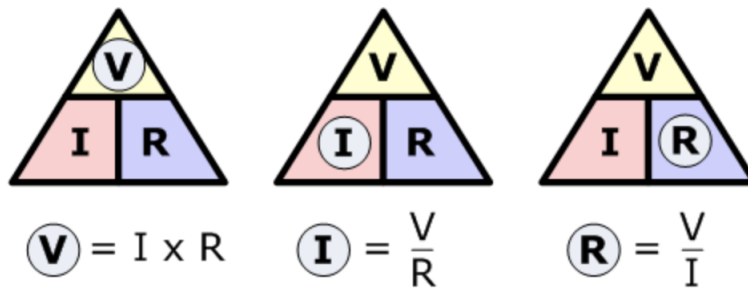
_____ is a property by which the passage of current is opposed. _____ causes some of the electric energy to be transformed into heat. It is represented by "R" and measured in ohms (Ω). A light bulb is an example of a _____.

Ohm's Law

Georg Ohm was a German mathematician and physicist who discovered an important relationship between voltage, current, and resistance. **Ohm's Law** states that the amount of voltage across a circuit is proportional to the amount of current running through it. This proportionality is called the resistance.

Recall that:

- Voltage (V) is measured in Volts (V) and comes from batteries.
- Resistance (R) is measured in Ohms (Ω) and is caused by a block in the wire.
- Current (I) is measured in amps (A) which is the electrical current.



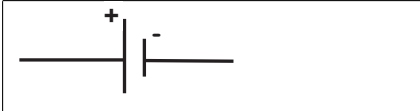


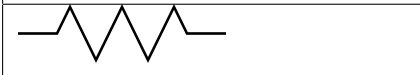
Retrieved from: <http://www.electronics-tutorials.ws/dccircuits/dcp2.html>

We can use Ohm's law to find the current, resistance, or voltage of any circuit if we are given the other two values.

Example:

In a simple circuit, there is a 20 volt battery and a light bulb with a resistance of 4 ohms. What is the electrical current running through this circuit?

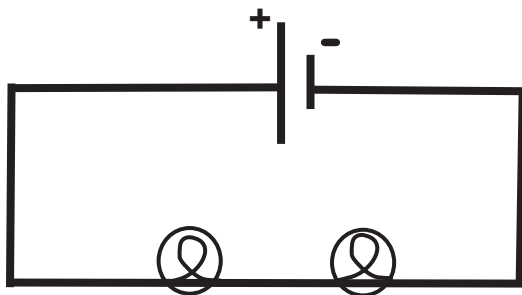
Common Symbols:

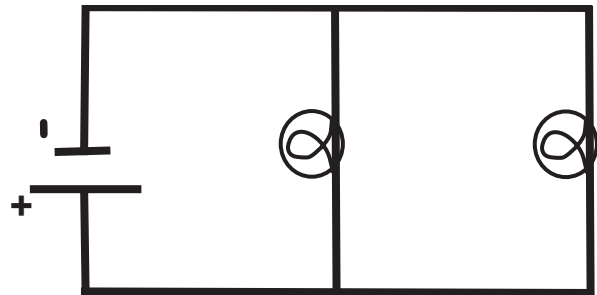
	Battery
	Light Bulb
	Switch
	Resistor

Types of Circuits

There are two ways in which light bulbs (or other resistors) can be arranged in one circuit. The two types are **Parallel** and **Series** circuits, each type having their own special properties.

Can you tell what type of circuits these are?



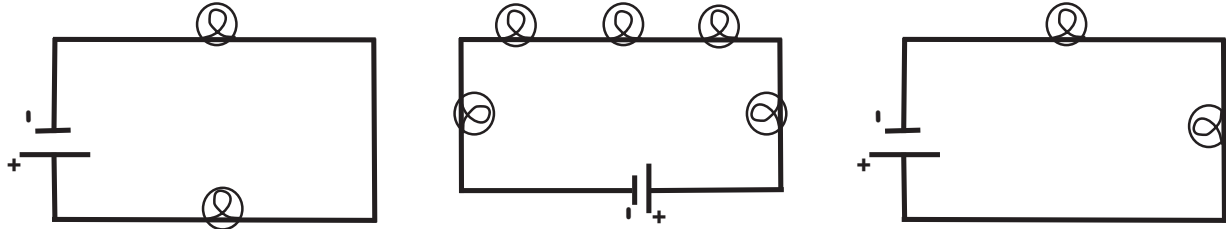


In a _____ circuit, the light bulbs run along one single path and are placed beside each other.

In a _____ circuit, the light bulbs run along different paths and are _____ to each other.

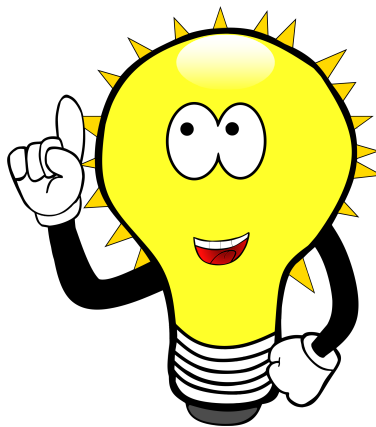
Series Circuits

Each of the following diagrams are examples of how light bulbs can be arranged in a series circuit.



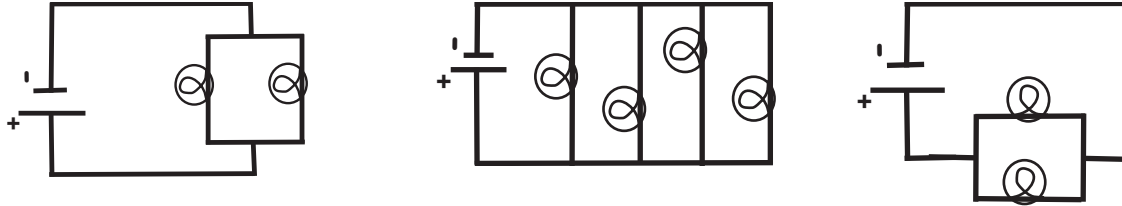
Properties

1. The total current in the circuit is equal to the current flowing through each resistor.
(ie. $I_{total} = I_1 = I_2 = \dots$)
2. The total voltage in the circuit is equal to the sum of the voltage at each resistor.
(ie. $V_{total} = V_1 + V_2 + \dots$)
3. The total resistance in the circuit is equal to the sum of the resistance at each resistor.
(ie. $R_{total} = R_1 + R_2 + \dots$)
4. If the circuit is broken at anytime (ie. A light bulb burns out, a switch is opened, or a wire is cut) the entire circuit _____.
5. If you add a light bulb to a series circuit the brightness of the bulbs will _____.
Adding a light bulb increases the resistance and reduces the current causing the lights to be _____.



Parallel Circuits

Each of the following diagrams are examples of how light bulbs can be arranged in a parallel circuit.



Properties

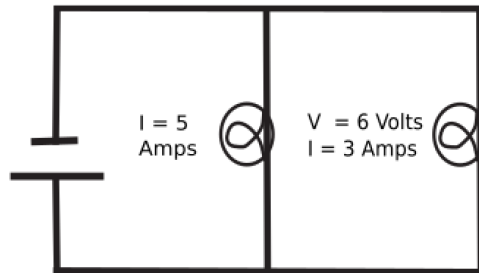
1. The total current in the circuit is equal to the sum of the current flowing through each of the resistors in the circuit.
(ie. $I_{total} = I_1 + I_2 + \dots$)
2. The total voltage in the circuit is equal to the voltage at each of the resistors in the circuit.
(ie. $V_{total} = V_1 = V_2 = \dots$)
3. The inverse of the total resistance in the circuit is equal to the sum of inverses of the resistance at each of the resistors.
(ie. $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$)
4. If one of the paths break (ie. a light bulb burns out, a switch is opened, or a wire is cut) the rest of the circuit will _____.
5. If you add a light bulb to a parallel circuit the brightness of the bulbs will _____
_____. *Since each light bulb has its own path and therefore does not share voltage, the brightness _____.*



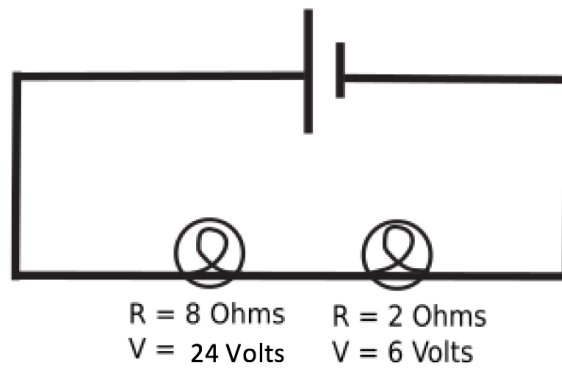
Examples

Determine if the circuit is parallel or series, then find the total current, voltage, and resistance of the circuit.

1.

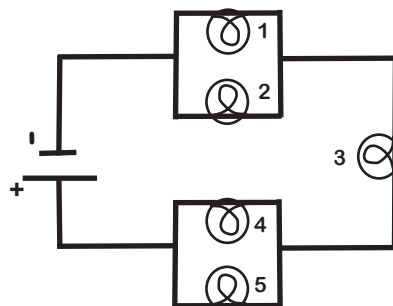
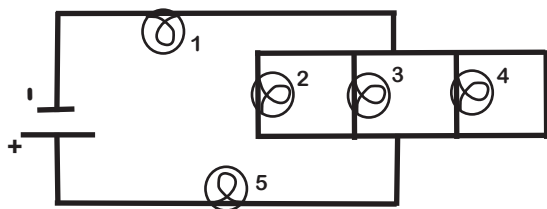


2.



Series and Parallel Circuits

In some cases, a combination of both parallel and series circuits may be used. The following images are examples of what it would look like to have a parallel-series circuit.



How to Approach Combination Questions

1. Treat the parallel component as one single resistor in the series component
2. Solve each component individually

Note: The values for current, voltage, and resistance of the single resistor will be the total values of the parallel component.

Example

Referring to the first diagram above, given that light bulb 1 has 4 volts, light bulb 5 has 6 volts and the total voltage for the circuit is 19 volts, how many volts does light 4 have?

Example

Referring to the second diagram above, given that light bulb 2 has a current of 5 amps, light bulb 3 has a current of 8 amps and light bulb 5 has a current of 7 amps, what is the current at light bulbs 1 and 4?

What is the total current?

Advantages and Disadvantages of Series and Parallel Circuits

You may be wondering when it is more beneficial to have a series circuit over a parallel circuit - or vice versa, so here are some advantages and disadvantages:

Advantages of Series

- They do not overheat
- Easy to learn and make
- We can add more power devices (increase specific voltages)
- All resistors carry the same current

Advantages of Parallel

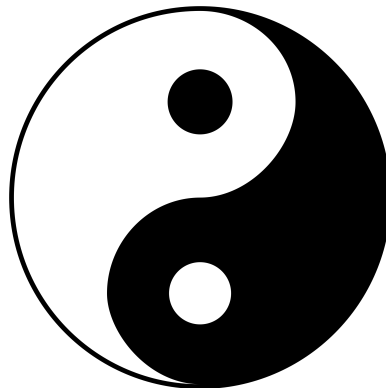
- Each resistor gets the same amount of voltage
- Easy to add or replace resistors without impacting the rest of the circuit
- The current is able to flow through multiple paths

Disadvantages of Series

- If there is one break, the entire circuit stops working
- The more resistors you add the greater the resistance will be

Disadvantages of Parallel

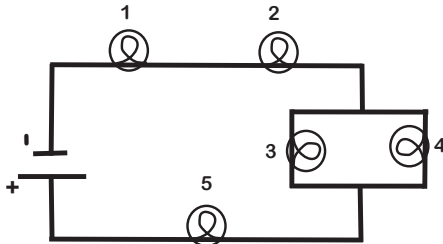
- Requires a lot of wires (expensive)
- Cannot increase the voltage of specific resistors



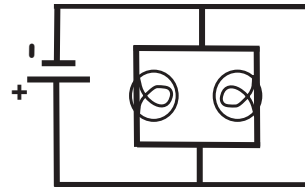
Problem Set

1. For each of the diagrams below, determine if the light bulbs are in a series, parallel, or parallel-series circuit.

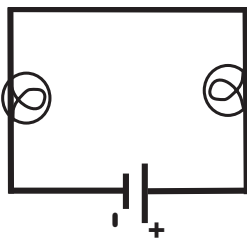
a)



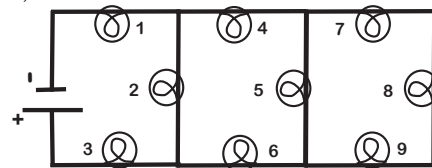
b)



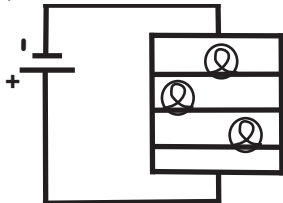
c)



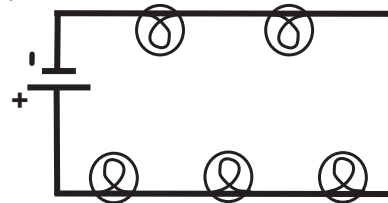
d)



e)



f)



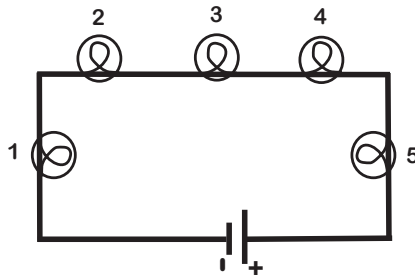
2. Ryan is installing lights in his garden to help show off his plants. Ryan however, isn't a very good gardener and so he has a lot of very dry plants. Should Ryan use a parallel or series circuit when installing his lights? Explain.

3. Sally is making a series circuit with light bulbs and batteries. When she connects 1 light bulb with 2 batteries it comes on with normal brightness. Fill out the table below to see what happens when Sally uses a different amount of light bulbs and batteries.

Number of Light Bulbs	Number of Batteries	Brightness (Dim, normal, bright)
1	2	
2	2	
2	4	
2	6	
3	6	

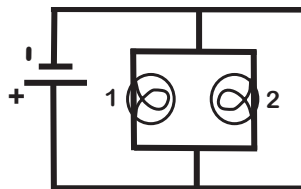
4. For each of the diagrams and corresponding tables below, fill in the missing values in the table using Ohms Law and circuit properties.

a)



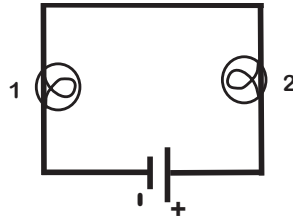
$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 = 12V$	$V_4 =$	$V_5 = 4V$
$I_{total} =$	$I_1 =$	$I_2 =$	$I_3 = 6A$	$I_4 =$	$I_5 =$
$R_{total} =$	$R_1 = 2\Omega$	$R_2 = 6\Omega$	$R_3 =$	$R_4 = \frac{1}{3}\Omega$	$R_5 =$

b)



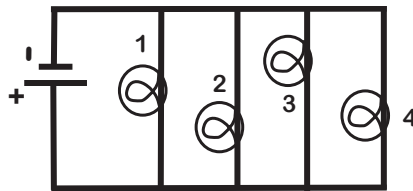
$V_{total} =$	$V_1 =$	$V_2 = 5V$
$I_{total} =$	$I_1 = 3A$	$I_2 =$
$R_{total} =$	$R_1 =$	$R_2 = 4\Omega$

c)



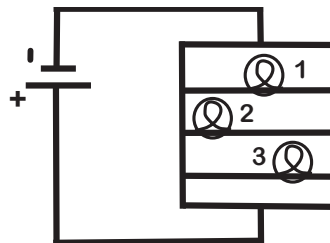
$V_{total} =$	$V_1 = 8V$	$V_2 =$
$I_{total} = 0.5A$	$I_1 =$	$I_2 =$
$R_{total} =$	$R_1 =$	$R_2 = 4\Omega$

d)



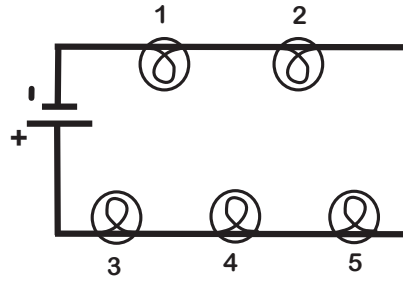
$V_{total} =$	$V_1 = 11V$	$V_2 =$	$V_3 = 11V$	$V_4 =$
$I_{total} =$	$I_1 =$	$I_2 =$	$I_3 =$	$I_4 =$
$R_{total} =$	$R_1 = 1\Omega$	$R_2 = 2\Omega$	$R_3 = 3\Omega$	$R_4 = 6\Omega$

e)



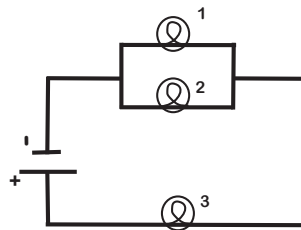
$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 =$
$I_{total} =$	$I_1 =$	$I_2 =$	$I_3 = 8A$
$R_{total} = 4\Omega$	$R_1 = 8\Omega$	$R_2 =$	$R_3 = 16\Omega$

f)



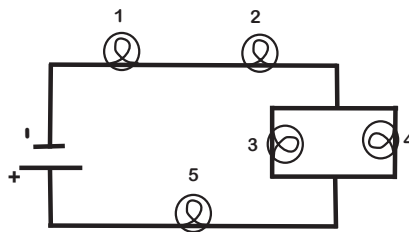
$V_{total} =$	$V_1 =$	$V_2 = 16V$	$V_3 =$	$V_4 =$	$V_5 = 28V$
$I_{total} =$	$I_1 =$	$I_2 = 4A$	$I_3 =$	$I_4 =$	$I_5 =$
$R_{total} = 25\Omega$	$R_1 = 3\Omega$	$R_2 =$	$R_3 = 5\Omega$	$R_4 =$	$R_5 =$

g)



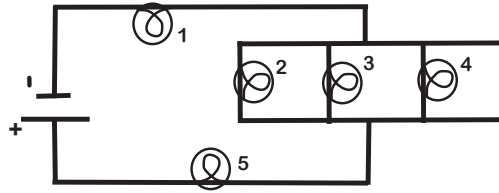
$V_{total} =$	$V_1 =$	$V_2 =$	$V_3 = 12V$
$I_{total} = 4A$	$I_1 =$	$I_2 =$	$I_3 =$
$R_{total} =$	$R_1 = 12\Omega$	$R_2 = 4\Omega$	$R_3 =$

h)



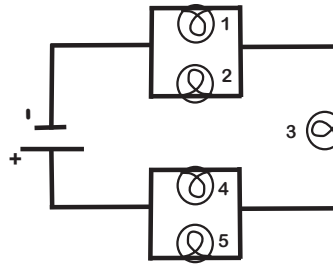
$V_{total} =$	$V_1 =$	$V_2 = 3V$	$V_3 =$	$V_4 = 6V$	$V_5 =$
$I_{total} = \frac{3}{2}A$	$I_1 =$	$I_2 =$	$I_3 = 1A$	$I_4 = \frac{1}{2}A$	$I_5 =$
$R_{total} =$	$R_1 = 4\Omega$	$R_2 =$	$R_3 =$	$R_4 =$	$R_5 = 2\Omega$

i)



$V_{total} = 154V$	$V_1 = 132V$	$V_2 =$	$V_3 =$	$V_4 =$	$V_5 = 10V$
$I_{total} =$	$I_1 =$	$I_2 =$	$I_3 =$	$I_4 =$	$I_5 =$
$R_{total} =$	$R_1 = 6\Omega$	$R_2 = 3\Omega$	$R_3 = 2\Omega$	$R_4 = 1\Omega$	$R_5 = \frac{5}{11}\Omega$

j)



$V_{total} =$	$V_1 = 8V$	$V_2 = 8V$	$V_3 = 12V$	$V_4 = 5V$	$V_5 = 5V$
$I_{total} = 10A$	$I_1 =$	$I_2 = 2A$	$I_3 =$	$I_4 = 8A$	$I_5 =$
$R_{total} =$	$R_1 =$	$R_2 =$	$R_3 =$	$R_4 =$	$R_5 =$

k) Repeat i) assuming light bulb 4 is burnt out

$V_{total} = 154V$	$V_1 = 132V$	$V_2 =$	$V_3 =$	$V_4 = N/A$	$V_5 = 10V$
$I_{total} =$	$I_1 =$	$I_2 =$	$I_3 =$	$I_4 = N/A$	$I_5 =$
$R_{total} =$	$R_1 = 6\Omega$	$R_2 =$	$R_3 =$	$R_4 = N/A$	$R_5 = \frac{5}{11}\Omega$

l) Repeat j) assuming light bulb 2 is burnt out

$V_{total} =$	$V_1 = 8V$	$V_2 = N/A$	$V_3 = 12V$	$V_4 = 5V$	$V_5 = 5V$
$I_{total} = 10A$	$I_1 =$	$I_2 = N/A$	$I_3 =$	$I_4 = 8A$	$I_5 =$
$R_{total} =$	$R_1 =$	$R_2 = N/A$	$R_3 =$	$R_4 =$	$R_5 =$