



Grades 7 & 8, Math Circles

27/28 February, 1 March, 2018

Mathematical Magic

Card Tricks

Have you ever seen a magic show? Do you know of any cool card tricks? Well chances are, the tricks you have seen are all created on the basis of simple mathematical counting techniques. Today we are going to take on the role of mathematical magicians, learning cool card tricks using math!

Final Three

The Final Three is a card trick that allows the audience member to pick three cards, and after a series of mathematical tricks ends up finding those three cards.

How it Works

1. Have the spectator pick three cards, or three people each pick one card
2. Make 4 piles of 10, 15, 15 and then 9 (the rest of the cards) face down
3. Place the first selected card face down on top of the first pile that has 10 cards
4. Have the spectator take a cut from the second pile and place it on top of first pile
5. Place the second card face down on the second pile
6. Have the spectator take a cut from the third pile and place it on top of the second pile
7. Place the third card face down on the third pile and place the leftover 9 cards face down on top
8. Pick up the three piles in reverse order so that pile 3 is on top, pile 2 is in the middle, and pile 1 is on the bottom
9. Take off the top 4 cards and place them on the bottom
10. Now turn the first card face up and second face down in separate piles

11. Continue through the entire deck and have them stop you when they see their card
 - They will never see their card
12. Repeat this pattern with the face down pile until you are left with three cards
 - The three cards will be the original cards

Why it Works

- When you complete steps 10-12 (the process of flipping alternating cards face up and face down into separate piles) the final three cards you are left with are always the cards in the 6th, 22nd, and 38th place
 - When you go through the deck the first time, you flip every card in an odd position face up, creating a new face down pile that contain the cards that were in the 52nd, 50th, 48th, 46th, 44th, 42nd, 40th, 38th, 36th, 34th, 32nd, 30th, 28th, 24th, 26th, 22nd, 20th, 18th, 16th, 14th, 12th, 10th, 8th, 6th, 4th, and 2nd positions
 - Eliminating every other card on your second run through creates a new face down pile containing the cards that were in the 2nd, 6th, 10th, 14th, 18th, 22nd, 26th, 30th, 34th, 38th, 42nd, 46th and 50th positions
 - The third time leaves your new face down pile with the cards that were in the 46th, 38th, 30th, 22nd, 14th, and 6th positions
 - One final time leaves you with three cards that were in the 6th, 22nd, and 38th position
 - Notice that each time we simply eliminated the cards in the odd positions
- Having piles of 10, 15, 15, and 9 as well as the way the piles are stacked ensures that the 1st and 2nd and also the 2nd and 3rd selected cards have 15 cards between them.
- After the spectator places the first selected card on the pile of 10 cards, it does not matter how many cards they cut from the next pile because when you pick the four piles up, there will always be 15 cards between the selected cards.
- Once the piles have been picked up, one of the selected cards is in the 10th position. Thus, by moving the top 4 cards to the bottom you move it into the 6th position.
- Since the selected cards are 15 apart, it ends up that the selected cards are in the 6th, $7 + 15 = 22$ nd, and $23 + 15 = 38$ th positions.

Cross-Section

The cross-section card trick will leave your audience stumped as to how you discovered which card is theirs.

How it Works

1. Lay out 16 cards in a 4x4 set up
2. Have the spectator pick a card and only reveal which column it is in
3. Pick the cards up column by column and then relay the cards again in a 4x4 layout where each column from before is now a row
 - Column 1 becomes row 1
 - Column 2 becomes row 2
 - Column 3 becomes row 3
 - Column 4 becomes row 4
4. Ask what column their card is in again and you will know which card is theirs
 - This is because you know what row their card is in from step (2), so by asking what column their card is in now, you will see which of the 4 cards from that row is theirs
5. As you mix all the cards together, discretely find their card and place it on the top or bottom of the pile (as long as you know where it is it does not matter)
6. Make groups of 4 cards face down making sure you know where their card is
7. Ask them to pick 2 of the 4 groups
 - If one of the groups they chose has their card in it, keep both groups they picked and remove the other two
 - If both groups do not have their card in them, discard both of them
8. Ask the spectator to then pick 2 cards from each group
 - If one of the cards they chose is their card, keep all four cards (two from each group) and remove the other two cards from each group
 - If neither card is their card, discard all four cards they picked

9. Now ask the spectator to pick 1 card from each group

- If they pick their card, keep both cards they picked and remove the other two cards
- If they do not pick their card, discard the two cards they picked

10. Finally, have the spectator pick one card

- If they pick their card, turn it over
- If they do not pick their card, discard it and turn over their card

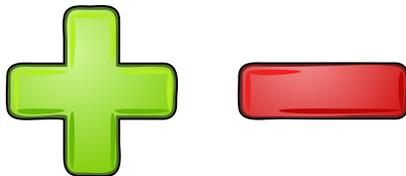


Sum of 10

This trick strictly uses addition and subtraction to calculate the cards that the spectator has chosen and in turn will amaze the audience.

How it Works

1. Have the spectator pick two cards from the deck.
 - The two cards cannot add up to 10 and they cannot chose a face card or a 10.
 - Aces are worth one, face cards are worth 10
 - Let the spectator keep the cards
2. Start turning the rest of the cards face-up, if you turn up a 10 or a face card immediately place another card on top it.
3. If you do not flip over a 10 or a face card, start a new pile
4. As you make new piles, if there are ever two cards on the top of any piles that add up to 10, stop and place two new cards on top of both of those piles
5. Repeat until you run out of cards
 - Note: You can make as many piles as required
6. Once you run out of cards you must remove any additional piles that add up to 10 together or that have a face card and/or a 10 on top
7. You should be left with two piles. In your head subtract 10 from each card on the top of the piles and those two values will be the values of the two cards the spectator picked



Why it Works

- Notice that the entire deck can be split into various combinations of 10.

Four 10's → Worth 10

$$5 + 5 = 10$$

Four Kings's → Worth 10

$$6 + 4 = 10$$

Four Queens's → Worth 10

$$7 + 3 = 10$$

Four Jacks's → Worth 10

$$8 + 2 = 10$$

$$9 + \textit{Ace} = 10$$

- Therefore, since the spectator picks two cards that do not total 10, there will always be two cards leftover that cannot total ten. This is because the matching card that is needed to make ten is in the hands of the spectator.
- Since we cancel out any combination that adds to 10 we will always be able to eliminate all piles except for 2 of them
- For example if they selected a 7 and 4, the matches 3 and 6 will be the two cards left over and every other card will be able to match with its partner.

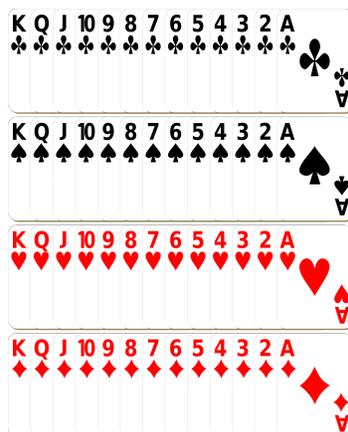


Match and Count

This multi-step card trick allows a spectator to pick a card from a subset of cards and have the magician find out where their card is in the deck.

How it Works

1. Give the spectator any 9 cards and have them pick just one of those cards
2. Place this card on top of the other 8 cards
3. Place the rest of the deck (43 cards) on top of this pile
4. Make 4 piles where you flip over cards from the deck as you count down from 10 each time you flip. You stop making a pile if you *get a match*
 - A match is when you say a certain number and actually flip over a card with that same value
 - For example if you say 10, 9, 8 and actually flip over an 8 as your third card
 - Note face cards are considered 10 and aces are considered 1
5. If there is no match when you get to one, place an additional card face down on top of that pile to “cap it off”
6. Add up the value of the cards on top of the four piles
 - Capped piles are worth zero
7. Then count that many cards down from the leftover cards and that will be their card
 - If there are no matches on any pile then the last card, the card you would use to cap off the 4th pile is their card

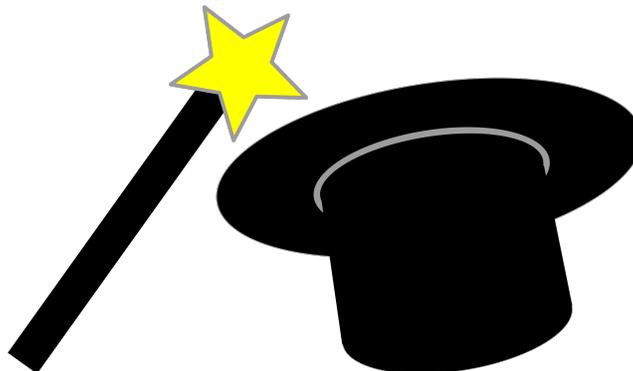


Split Stack

Split Stack gives the spectator all of the power and makes them seem as if they are controlling the entire trick.

How it Works

1. Split the deck into 2 piles of 26 cards
 - Let's call them pile A and B
2. Have the spectator cut pile A (wherever they desire) and call this new pile C
3. Then let the spectator pick one card from pile B and place this card on the bottom of the pile B
 - If comfortable you may shuffle the cards but keep the card they picked on the bottom
4. Place pile B on top of the remaining cards in pile A, and call this new pile D
5. Do the Klondike shuffle on pile D
 - This is where you take the top and bottom card of pile D each time and place them face down on the table
 - Note: Order matters, keep the top card on the top and bottom card on the bottom
6. Count the amount of cards in pile C
 - This will be the position of the spectator's card in pile D
 - Therefore, take pile D and count out the number of cards that was in pile C and you will turn over their card.



The Logic Trick

This sneaky trick combines math, memorization, and story telling into a card trick that your audience will never be able to figure out on their own.

How it Works

1. Count out 26 cards to split the deck into 2 piles, A and B.
 - Count the 26 cards face up
 - Memorize the 7th card as it goes by (This will be the 7th card of pile A)
2. Then with the pile B make three new piles of “10”
3. To make the piles flip the first card face up and count the rest of the way to 10 by placing cards face down on top of the first card
 - Ex. If you turn over a 7 then place 3 cards face down on top of the 7 since $7 + 3 = 10$
 - Face cards are equal to 10 and aces are equal to one, if you flip over a 10 as the first card then move on to the next pile
4. Place the rest of the cards from pile B that were not used to make the three piles on top of pile A, face down and call this pile C
5. Add the value of the 3 face up cards and that will be the position of the 7th card that you memorized in pile C
6. Make up a story as to why the card in that position is the one that it is
 - Ex. If you have a 3 of hearts in the 7th position and 2, 4, 5 of hearts as your face up cards, say the only one missing is the 3 of hearts

Sneaky Memorization

This one requires you to have a good memory and fast reflexes in order to truly impress your audience.

How it Works

1. Memorize the card on the bottom of the deck
2. Have the spectator pick any card (but the bottom one)
3. Make three piles of roughly the same amount, keeping in mind where the pile with the known bottom card is
4. Have them place their card on any of the three piles
5. If they place it on either one of the piles with an unknown bottom card, place the pile with the known bottom card on top of it and the last pile on the bottom (This ensures the card you know is on top of their card)
6. If they put it on the same pile where the bottom card is known, move the bottom card to the top before combining the three piles
7. Begin to make piles of about 5-6 cards face up, keeping an eye out for the card that you know
8. Once you turn over your card immediately start a new pile
9. You will see what their card is and you can memorize it while you finish making that pile
10. Then on your next pile spell out their card (in your head) and place that many cards in that pile
 - Ex. 2 of clubs would need 10 cards since "Two of clubs" has 10 letters
11. Make a few more piles with the remaining cards that you have
12. Pick up the pile with the number of cards needed to spell their card first, then place the pile with their card under it and then the rest of them in any order
13. Ask what their card is and spell it out and the next card you turn over is theirs!

Subsets

Get ready to count a lot, in this trick there is a mathematical sequence required to identify which card is the spectator's.

How it Works

1. Have the spectator pick any number of cards (less than 52) and discard the rest
2. Have them pick one card from this new pile, call it A
3. Place that card on the bottom and shuffle but keep that card on the bottom
4. Have them choose a number that is greater than half of the number of cards in pile A but less than the total number of cards in pile A
 - Ex. If pile A has 12 cards, they can pick 7, 8, 9, 10 or 11
5. Count out this many cards and place the remaining ones on top
 - Ex. If pile A has 12 cards, and they pick 9, count out 9 cards face down and place the additional 3 cards on top
6. Repeat twice more and on your last try the top card on pile A will be theirs

Why it Works

- Let's explore why this trick works with the example where pile A has 12 cards and the spectator has selected 9 as their number
- Before we count out 9 cards, let's give each card a name: $a, b, c, d, e, f, g, h, i, j, k, l$. We know that their card is in the 12th position (at the bottom of the deck) so their card is "l"
- After we count out 9 cards and place the remaining 3 on top, their card is now in the 3rd position, $j, k, l, i, h, g, f, e, d, c, b, a$
- Counting out 9 more cards and placing the extra 3 on top again puts the spectator's card in the 10th position, $c, b, a, d, e, f, g, h, i, l, k, j$
- Finally after the 3rd time, their card has moved to the 1st position, hence it is on top of the pile, $l, k, j, i, h, g, f, e, d, a, b, c$

Down or Switch

This fun little trick gives the power to the spectator in that they have complete control how the cards are mixed up leaving them stunned when you tell them the answer.

How it Works

1. Take the cards Ace through 9 of any suit, and make sure they are in order (Ace, 2, 3, 4, 5, 6, 7, 8, 9) and discard all other cards
2. Tell the audience they will be allowed to decide if each card is going to be put down normally or switched with the next card
3. While explaining this turn the Ace face-up and place it on the table
4. Next turn the two face-up and put it on top of the ace
5. Switch the 3 and 4 so that you put the 4 face-up on top of the two, followed by the 3
6. Next put down the 5 face-up on top of the 3
7. Switch the 6 and 7 so that the 7 is on the 5 and the 6 is on the 7
8. Place the the 8 on top of the 6 and then the 9 on top of the 8
 - Note: All cards should have been placed face-up
9. Pick up the pile, turn it over and explain to the audience once more they can choose to put the card down (put down the Ace face-up) or switch the cards (switch the 2 and the 4 so that the 4 is on top of the Ace and the 2 on top of the 4)
10. Stop and pick these three cards up and place them at the bottom of the pile
 - Note: All cards are now face down in the pile
11. Now let the spectator begin to choose, but this time do not turn the cards face up when they say switch or put down
 - They can go through the deck as many times as they want

12. After each time they go through the deck, the cards will always be in the same order
- If they go through the deck an even number of times the cards will be in the regular order, 3, 5, 7, 6, 8, 9, Ace, 4, 2
 - Lay the cards out from left to right when you reveal
 - If they go through an odd number of times the cards will be in reverse order, 2, 4, Ace, 9, 8, 6, 7, 5, 3
 - Lay the cards out from right to left when you reveal
 - Therefore you can say “There are 9 cards here, and you told me how to organize the cards, so there is no way I could know the order of these cards. Furthermore, since there are 9 cards, that means there are 362,880 different ways these cards could be arranged. However, I think your cards are in the order 357,689,142
 - Note: Ace represents 1 in this case

Can you figure out why this always works?

