



Grades 7 & 8, Math Circles

6/7/8 February, 2018

Warm-Up Solutions

Welcome back Mathematicians! Before we begin today's lesson, let's start by doing some warm up questions to get our brains ready to learn. *Complete all problems without a calculator.*

Warm-Up

1.	$\begin{array}{r} 37 \\ \times 25 \\ \hline 925 \end{array}$	2.	$\begin{array}{r} 82 \\ \times 9 \\ \hline 738 \end{array}$	3.	$\begin{array}{r} 233 \\ \times 11 \\ \hline 2563 \end{array}$	4.	$\begin{array}{r} 15 \\ \times 34 \\ \hline 510 \end{array}$	5.	$\begin{array}{r} 158 \\ \times 25 \\ \hline 3950 \end{array}$
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6.	$\begin{array}{r} 97 \\ \times 17 \\ \hline 1649 \end{array}$	7.	$\begin{array}{r} 21 \\ \times 12 \\ \hline 252 \end{array}$	8.	$\begin{array}{r} 605 \\ \times 21 \\ \hline 12705 \end{array}$	9.	$\begin{array}{r} 43 \\ \times 70 \\ \hline 3010 \end{array}$	10.	$\begin{array}{r} 337 \\ \times 111 \\ \hline 37407 \end{array}$
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- | | |
|---|--|
| 11. Is 23,121 divisible by 9? Yes | 13. Is 8,687,465,684,135 divisible by 2? No |
| 12. Is 16,777,216 divisible by 3? No | 14. Is 2,985,984 divisible by 12? Yes |

Evaluate.

- | | |
|-----------------------------------|------------------------------------|
| 15. $57^2 = \underline{3249}$ | 19. $225^2 = \underline{50625}$ |
| 16. $345 \div 5 = \underline{69}$ | 20. $635 \div 5 = \underline{127}$ |
| 17. $97^2 = \underline{9409}$ | 21. $135^2 = \underline{18225}$ |
| 18. $475 \div 5 = \underline{95}$ | 22. $115 \div 5 = \underline{23}$ |



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Mental Math Solutions

Mental Math Keeps You Sharp!

Today we are going to look at several different mathematical tricks that will help improve your mental math skills. As mathematicians it is important to keep up with our mental math skills to be able to solve problems quickly and efficiently. By the end of today you will be equipped with new tricks you can use to impress your friends and family! Let's begin by warming up our brains with some questions.

Multiplication Tricks

Multiplying with 5

This first trick will allow you to multiply large numbers by 5 very quickly.

1. Take the number you are multiplying 5 by and divide it by 2
2. Add zero to the end of that number!

Note: If there is a decimal, instead of adding a zero, just multiply by 10, or move the decimal over one place.

Examples:

Evaluate 428×5 .

$428 \div 2 = 214$. Adding a zero gives 2140

Evaluate 45×5 .

$45 \div 2 = 22.5$. Multiplying by 10 gives 225

Try it yourself!

$$67 \times 5 =$$

$$67 \div 2 = 33.5$$

$$= 335$$

$$178 \times 5 =$$

$$178 \div 2 = 89$$

$$= 890$$

$$2463 \times 5 =$$

$$2463 \div 2 = 1231.5$$

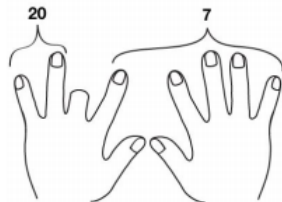
$$= 12315$$

Multiplication by 9

1. Put both your hands in front of you.
2. When you are asked to multiply a number from 1-10 by 9, you put that corresponding finger down (Let your pinky on your left hand represent 1, and your pinky on your right hand represent 10) *Include your thumbs as well!*
3. The number of fingers to the left of the one you put down is the tens digit of the answer.
4. The number of fingers to the right of the one you put down is the ones digit of the answer.

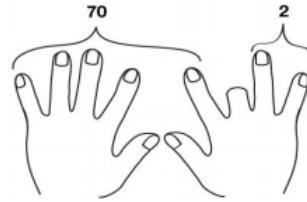
Remember: Whatever number you want to multiply by nine, that's the finger you fold down.

If you wanted to multiply 9×3 , your fingers would look like this:



$$9 \times 3 = 27$$

If you wanted to multiply 9×8 , your fingers would look like this:



$$9 \times 8 = 72$$

Retrieved from <https://www.superteacherworksheets.com/multiplication/nines-trick.pdf>

Now you might be thinking, what about multiplying large numbers by nine! Well luckily there is a simple trick to solve these questions too.

1. Place a zero at the end of the number you are multiplying by 9
2. Subtract the original number from this new number

Example:

$$47 \times 9$$

→ Adding a zero to the end gives 470

$$\rightarrow 470 - 47 = 423$$

That's it 423, is our answer!

Why does this work you ask?

Consider any random number, call it n .

$$\text{Then } n \times 9 = n \times (10 - 1)$$

$$= (n \times 10) - (n \times 1)$$

$$= (n \times 10) - n$$

Try it yourself!

$67 \times 9 =$

670

$670 - 67 = 603$

$178 \times 9 =$

1780

$1780 - 178 = 1602$

$463 \times 9 =$

4630

$4630 - 463 = 4167$

General Two Digit Multiplication Tip

The following diagram reviews a simple way to calculate the multiplication between two digit numbers.

Step 1

$$\begin{array}{r} 38 \\ \downarrow \\ 72 \\ \hline 6 \end{array}$$

Multiply the ones digits.
 $= 8 \times 2$
 $= 16$
Carry over the one.

Step 2

$$\begin{array}{r} 38 \\ \swarrow \searrow \\ 72 \\ \hline 36 \end{array}$$

Multiply the ones and tens digits.
Add them with the carry over.
 $= (3 \times 2) + (8 \times 7) + 1$
 $= 63$.
Carry the 6.

Step 3

$$\begin{array}{r} 38 \\ \downarrow \\ 72 \\ \hline 2736 \end{array}$$

Multiply the tens digits, add it to the carry over.
 $= (3 \times 7) + 6$
 $= 27$

Try it yourself!

$97 \times 24 = 2328$

$64 \times 41 = 2624$

$68 \times 29 = 1972$

Multiplication by 11

What happens when you forget your calculator at home and you are asked to multiply 42 by 11, what do you do!? Well, you probably guessed it, there is an easy way to solve two digit multiplication by 11

This is best explained with an example:

1. Write the number, 42 as $4(\)2$, where the $(\)$ will be filled with a new number
2. The middle digit is the sum of the first two digits, $4 + 2 = 6$
3. The answer is $4(6)2 = 462$

Note Sometimes the sum of the first two numbers is greater than 9. In that case you simply add one to the first digit and keep the second digit for the middle number.

Example:

$$\begin{aligned}47 \times 11 \\ \rightarrow 4(\)7 \\ \rightarrow 4 + 7 = 11 \\ \rightarrow 4(11)7 \\ \rightarrow (4 + 1 = 5)(1)7 \\ \rightarrow 517\end{aligned}$$

Try it yourself!

$36 \times 11 =$	$91 \times 11 =$	$71 \times 11 =$
$= 3(6 + 3)6$	$= 9(9 + 1)1$	$= 7(7 + 1)1$
$= 396$	$= 1001$	$= 781$

There is also a useful technique for solving multiplication of large numbers by 11.

1. Take the number and place a zero at the end of it
2. Add the original number and this new number together to get the answer.

Example:

468×11

$\rightarrow 4680$

$\rightarrow 4680 + 468 = 5148$

Try it yourself!

$957 \times 11 =$

9570

$= 9570 + 957$

$= 10,527$

$3682 \times 11 =$

36820

$= 36820 + 3682$

$= 40,502$

$12768 \times 11 =$

127680

$= 127680 + 12768$

$= 140,488$

Multiplying with 15

This next math rule will help you compute multiplication with large numbers by 15 with incredible speed!

1. Write zero at the end of the number you have
2. Divide this by 2
3. Add the two numbers

Example:

63×15

\rightarrow Adding a zero to the end gives 630

$\rightarrow 630 \div 2 = 315$

$\rightarrow 630 + 315 = 945$

Here is why it works.

$(n \times 15) = n \times (10 + 5)$

$= (n \times 10) + (n \times 5)$

\rightarrow Since multiplying by 10 is the same as adding a zero to the end of the number and since $(n \times 10) \div 2 = (n \times 5)$ We get that it works for any n .

Try it yourself!

$18 \times 15 =$

$= 180 \div 2 = 90$

$= 180 + 90 = 270$

$56 \times 15 =$

$= 560 \div 2 = 280$

$= 560 + 280 = 840$

$125 \times 15 =$

$= 1250 \div 2 = 625$

$= 1250 + 625 = 1875$

Multiplying with 25

Here are the steps that will help you multiply large numbers by 25 very efficiently:

1. Add two zeros to the end of the number
2. Divide by two
3. Divide again by two and you're done!

Example:

43×25

→ Adding 2 zeros to the end gives 4300

$\rightarrow 4300 \div 2 = 2150$

$\rightarrow 2150 \div 2 = 1075$

Try it yourself!

$33 \times 25 =$

$= 3300 \div 2 = 1650$

$= 1650 \div 2 = 825$

$74 \times 25 =$

$= 7400 \div 2 = 3700$

$= 3700 \div 2 = 1850$

$110 \times 25 =$

$= 11000 \div 2 = 5500$

$= 5500 \div 2 = 2750$

Squaring a number that ends in 5

This handy trick will allow you to find the square of any number ending in 5 with just a few simple steps!

1. Ignore the 5 in the units place
2. Treat the digits in front of the 5 as one number, add one to it and then multiply the two numbers together
3. Finally, place the number 25 at the end of the product found in step (2.)

Example: Find 75^2 .

$$\rightarrow 7 + 1 = 8$$

$$\rightarrow 7 \times 8 = 56$$

$$\rightarrow \text{Thus, } 75^2 = 5625$$

Example: Find 645^2 .

$$\rightarrow 64 + 1 = 65,$$

$$\rightarrow 64 \times 65 = 4160. \text{ Thus } 645^2 = 416025$$

Try it yourself!

$$35^2 =$$

$$3 + 1 = 4$$

$$3 \times 4 = 12$$

$$= 1,225$$

$$125^2 =$$

$$12 + 1 = 13$$

$$12 \times 13 = 156$$

$$= 15,625$$

$$1005^2 =$$

$$= 100 + 1 = 101$$

$$100 \times 101 = 10100$$

$$= 1,010,025$$

Three Digit Multiplication

Here is a quick review on how to multiply three digit numbers without using a calculator. This method is easiest to understand using an example, so let's try 236×182 .

Step 1

$$\begin{array}{r} 236 \\ 182 \\ \hline 2 \end{array}$$

Multiply the ones digits together.
 $= 6 \times 2$
 $= 12$
 Carry the one

Step 2

$$\begin{array}{r} 236 \\ 182 \\ \hline 52 \end{array}$$

Multiply the ones and tens digits, and add it with the carry over.
 $= (2 \times 3) + (6 \times 8) + 1$
 $= 6 + 48 + 1 = 55$
 Carry the five

Step 3

$$\begin{array}{r} 236 \\ 182 \\ \hline 952 \end{array}$$

Multiply the ones by the hundreds, and the tens by each other. Add them with the carry over.
 $= (2 \times 2) + (6 \times 1) + (8 \times 3) + 5$
 $= 39$
 Carry the three

Step 4

$$\begin{array}{r} 236 \\ 182 \\ \hline 2952 \end{array}$$

Multiply the hundreds and tens digits. Add them with the carry over.
 $= (2 \times 8) + (1 \times 3) + 3$
 $= 22$
 Carry the two

Step 5

$$\begin{array}{r} 236 \\ 182 \\ \hline 42952 \end{array}$$

Finally multiply the hundreds digits and add them with the carry over.
 $= (2 \times 1) + 2$
 $= 4$

Try it yourself!

$111 \times 444 =$

$$\begin{aligned} &= (4 \times 1) = 4 \text{ Step 1} \\ &= (4 \times 1) + (4 \times 1) = 8 \text{ Step 2} \\ &= (4 \times 1) + (4 \times 1) + (4 \times 1) = 12 \text{ Step 3} \\ &= 1 + (4 \times 1) + (4 \times 1) = 9 \text{ Step 4} \\ &= (4 \times 1) = 4 \text{ Step 5} \\ &= 49,284 \end{aligned}$$

$642 \times 847 =$

$$\begin{aligned} &= (7 \times 2) = 14 \text{ Step 1} \\ &= 1 + (7 \times 4) + (2 \times 4) = 37 \text{ Step 2} \\ &= (3 + (6 \times 7) + (8 \times 2) + (4 \times 4) = 77 \text{ Step 3} \\ &= 7 + (6 \times 4) + (8 \times 4) = 63 \text{ Step 4} \\ &= 6 + (6 \times 8) = 54 \text{ Step 5} \\ &= 543,774 \end{aligned}$$

Squaring Any Number Quickly

1. Round your number to the nearest multiple of 10
2. Add the difference between the two numbers and call this 'a'
3. Subtract the difference between the two numbers and call this 'b'
4. Multiply the numbers found in (2.) and (3.) and add it to the square of the difference

Example: Find 168^2 .

- First round to 170
- Add the difference $168 + 2 = 170$
- Subtract the difference $168 - 2 = 166$
- Square the difference, $2^2 = 4$
- Multiply and add, $(170 \times 166) + 4 = 28224$

Try it yourself!

$$19^2 =$$

$$20$$

$$(19 + 1 = 20)$$

$$(19 - 1 = 18)$$

$$20 \times 18 = 360$$

$$360 + 1^2 = 361$$

$$797^2 =$$

$$800$$

$$(797 + 3 = 800)$$

$$(797 - 3 = 794)$$

$$800 \times 794 = 635,200$$

$$635,200 + 3^2 = 635,209$$

$$1012^2 =$$

$$= 1000$$

$$(1012 + 12 = 1024)$$

$$(1012 - 12 = 1000)$$

$$= 1000 \times 1024 = 1,024,000$$

$$1,024,000 + 12^2 = 1,024,144$$

Division Tricks

Dividing by 5

There are two simple steps that can help improve your arithmetic computation speed.

1. Multiply the number by 2
2. Move the decimal one spot to the left

Note This trick even works with decimals!

Example: Find $63 \div 5$.

$$\rightarrow 63 \times 2 = 126$$

\rightarrow Moving the decimal one to left gives 12.6

Let's Prove Why This Works

Note that moving the decimal place one to the left is equivalent to dividing by 10

$$= ((n \times 2) \div 10)$$

$$= \frac{(n \times 2)}{10}$$

$$= n \times \left(\frac{2}{10}\right)$$

$$= n \times \left(\frac{1}{5}\right)$$

$$= \left(\frac{n}{5}\right)$$

$$= n \div 5$$

Try it yourself!

$$154 \div 5 =$$

$$154 \times 2 = 308$$

$$= 30.8$$

$$84.25 \div 5 =$$

$$84.25 \times 2 = 168.5$$

$$= 16.85$$

$$79.5 \div 5 =$$

$$79.5 \times 2 = 159$$

$$= 15.9$$

Divisibility Rules The table below summarizes some helpful hints on how to check if a number is divisible by another number.

Divisible by:	Test:
2	The last digit in the number is even
3	The sum of all the digits is divisible by 3
4	The last two digits are divisible by 4
5	The number ends in 0 or 5
6	The number is divisible by 2 <i>and</i> 3
8	The last three digits are divisible by 8
9	The sum of the digits is divisible by 9
10	The number ends in 0
11	The <i>alternating</i> sum of the digits is divisible by 11 Notes: Start with subtraction. 0 is divisible by 11
12	The number is divisible by 3 <i>and</i> 4

Try it yourself!

- Is 123,456 divisible by 6?
Yes, because it is divisible by 2 and 3
- Is 1,657,444 divisible by 12?
No, because it is not divisible by 3
- Is 161,051 divisible by 11?
Yes, because the alternating sum,
 $1 - 6 + 1 - 0 + 5 - 1 = 0$ which is
divisible by 11
- Is 519,647 divisible by 2?
No, because the last digit is odd
- Is 94,632 divisible by 3?
Yes, because the sum of the digits,
 $9 + 4 + 6 + 3 + 2 = 24$, which is divisible
by 3
- Is 5,166,516,651,667,253 divisible by 9?
No, because the sum of the digits, 71, is
not divisible by 9

Problem Set

Complete all 55 problems without a calculator.

- | | | |
|-------------------------------|------------------------|------------------------|
| 1. Redo the Warm-Up | 19. 15×11 | 38. 11×40022 |
| 2. 85^2 | 20. 9×25 | 39. 397×207 |
| 3. 11×35 | 21. 25×25 | 40. 403^2 |
| 4. Is 725,148 divisible by 4? | 22. $6473.2958 \div 5$ | 41. 23×32 |
| 5. 236.5×5 | 23. 245187×25 | 42. 787×690 |
| 6. 15×67 | 24. 11×784 | 43. 1234×5678 |
| 7. 9×81 | 25. 3142×11 | 44. 63×54 |
| 8. 185×25 | 26. 32×99 | 45. 649957×11 |
| 9. Is 45,319 divisible by 3? | 27. 20^2 | 46. 53×24 |
| 10. $142.3 \div 5$ | 28. 548×381 | 47. 664×147 |
| 11. 135^2 | 29. 81×12 | 48. 964×102 |
| 12. $565 \div 5$ | 30. 2011^2 | 49. 79989^2 |
| 13. 25×57 | 31. 516×11 | 50. 56^2 |
| 14. 59×11 | 32. 325^2 | 51. 322×211 |
| 15. Is 34,272 divisible by 6? | 33. 45×67 | 52. 789×321 |
| 16. 5×1305 | 34. 122^2 | 53. 85^2 |
| 17. 9×25 | 35. 43×51 | 54. 11×3338 |
| 18. 25×15 | 36. 6784×1214 | 55. 83×74 |
| | 37. 16^2 | |

Problem Set Solutions

1. See Page 1
2. $85^2 =$
 $8 + 1 = 9$
 $8 \times 9 = 72$
 $= 7,225$
3. $11 \times 35 =$
 $3(3 + 5)5$
 $= 385$
4. Is 725,148 divisible by 4? Yes, both 4 and 8 are divisible by 4.
5. 236.5×5
 $236 \div 2 = 118.25$
 $= 1182.5$
6. 15×67 $670 \div 2 = 335$
 $670 + 335 = 1005$
7. 9×81
 $= 810 - 81$
 $= 729$
8. 185×25
 $= 18500$
 $= 18500 \div 2 = 9250$
 $= 9250 \div 2 = 4625$
9. Is 45,319 divisible by 3? Sum of the digits, 22 is not divisible by 3
10. $142.3 \div 5$
 $= 142.3 \times 2 = 284.6$
 $= 28.46$
11. 135^2
 $(135 + 5) = 140$
 $(135 - 5) = 130$
 $130 \times 140 = 18,200$
 $18,200 + (5^2)$
 $= 18,225$
12. $565 \div 5$
 $130 \times 2 = 1130$
 $= 113$
13. 25×57
 $= 5700$
 $= 5700 \div 2 = 2850$
 $= 2850 \div 2 = 1425$
14. 59×11
 $= 5(5 + 9)9$
 $= 5(14)9$
 $= (5 + 1)49$
 $= 649$
15. Is 34,272 divisible by 6? Yes, it is divisible by 2 and 3.
16. 5×1305
 $= 1305 \div 2 = 652.5$
 $= 6525$
17. 9×25
 $= 900$
 $= 900 \div 2 = 450$
 $= 450 \div 2 = 225$

OR

$$\begin{aligned} &= 250 \\ &= 250 - 25 \\ &= 225 \end{aligned}$$

$$\begin{aligned} 18. \quad &25 \times 15 \\ &= 1500 \\ &= 1500 \div 2 = 750 \\ &= 750 \div 2 = 375 \end{aligned}$$

OR

$$\begin{aligned} &= 250 \\ &= 250 \div 2 = 125 \\ &= 25 + 125 = 375 \end{aligned}$$

$$\begin{aligned} 19. \quad &15 \times 11 \\ &= 110 \\ &= 110 \div 2 = 55 \\ &= 55 + 110 = 165 \text{ OR} \\ &= 1(1 + 5)5 \\ &= 165 \end{aligned}$$

$$\begin{aligned} 20. \quad &9 \times 15 \\ &= 150 \\ &= 150 - 15 \\ &= 135 \\ \text{OR} \\ &= 90 \\ &= 90 \div 2 = 45 \\ &= 45 + 90 = 135 \end{aligned}$$

$$\begin{aligned} 21. \quad &25 \times 25 \\ &= 2500 \\ &= 2500 \div 2 = 1250 \\ &= 1250 \div 2 = 625 \end{aligned}$$

$$\begin{aligned} 22. \quad &6473.2958 \div 5 \\ &= 6473.2958 \times 2 = 12946.5916 \\ &= 1294.65916 \end{aligned}$$

$$\begin{aligned} 23. \quad &245187 \times 25 \\ &= 24518700 \\ &= 24518700 \div 2 = 12259350 \\ &= 12259350 \div 2 = 6,129,675 \end{aligned}$$

$$\begin{aligned} 24. \quad &11 \times 784 \\ &= 7840 \\ &= 7840 + 784 = 8624 \end{aligned}$$

$$\begin{aligned} 25. \quad &3142 \times 11 \\ &= 34120 \\ &= 34120 + 3412 = 37,532 \end{aligned}$$

$$26. \quad 32 \times 99 = 3168$$

$$27. \quad 20^2 = 400$$

$$28. \quad 548 \times 381 = 208,788$$

$$29. \quad 81 \times 12 = 972$$

$$\begin{aligned} 30. \quad &2011^2 (2011 + 11) = 2022 \\ &(2011 - 11) = 2000 \\ &2022 \times 2000 = 4,044,000 \\ &4,044,000 + (11^2) \\ &= 4,044,121 \end{aligned}$$

$$\begin{aligned} 31. \quad &516 \times 11 \\ &= 5610 \\ &= 5610 + 561 = 6171 \end{aligned}$$

$$\begin{aligned} 32. \quad &325^2 \\ &(32 + 1) = 33 \\ &32 \times 33 = 1056 \\ &= 105,625 \end{aligned}$$

$$33. \quad 45 \times 67 = 3015$$

$$\begin{aligned} 34. \quad &122^2 \\ &(122 + 22) = 144 \\ &(122 - 22) = 100 \end{aligned}$$

- $$144 \times 100 = 14,400$$
- $$14,400 + (22^2)$$
- $$= 14,884$$
35. $43 \times 51 = 2193$
36. $6784 \times 1214 = 8,235,776$
37. 16^2
- $$(16 + 6) = 22$$
- $$(16 - 6) = 10$$
- $$22 \times 10 = 220$$
- $$220 + (6^2)$$
- $$= 256$$
38. 11×40022
- $$400,220$$
- $$400,200 + 40,022$$
- $$= 440,242$$
39. $397 \times 207 = 82,179$
40. 403^2
- $$(403 + 3) = 406$$
- $$(403 - 3) = 400$$
- $$406 \times 400 = 162,400$$
- $$162,400 + (3^2)$$
- $$= 162,409$$
41. $23 \times 32 = 736$
42. $787 \times 690 = 543,030$
43. $1234 \times 5678 = 7,006,652$
44. $63 \times 54 = 3402$
45. 649957×11
- $$6,499,570$$
- $$6,499,570 + 649,957$$
- $$= 7,149,527$$
46. $53 \times 24 = 1272$
47. $664 \times 147 = 97,608$
48. $964 \times 102 = 98,328$
49. 79989^2
- $$(79989 + 11) = 80,000$$
- $$(79989 - 11) = 79,978$$
- $$80,000 \times 79,978 = 6,398,240,000$$
- $$6,398,240,000 + (11^2)$$
- $$= 6,398,240,121$$
50. 56^2
- $$(56 + 4) = 60$$
- $$(56 - 4) = 52$$
- $$60 \times 52 = 3120$$
- $$3120 + (4^2)$$
- $$= 3136$$
51. $322 \times 211 = 67,942$
52. $789 \times 321 = 253,269$
53. 85^2
- $$(8 + 1) = 9$$
- $$8 \times 9 = 72$$
- $$= 7225$$
54. 11×3338
- $$33,380$$
- $$33,380 + 3,338$$
- $$= 36,718$$
55. 83×74
- $$= 6142$$