Math Circles – Finite Automata
Question Sheet 1

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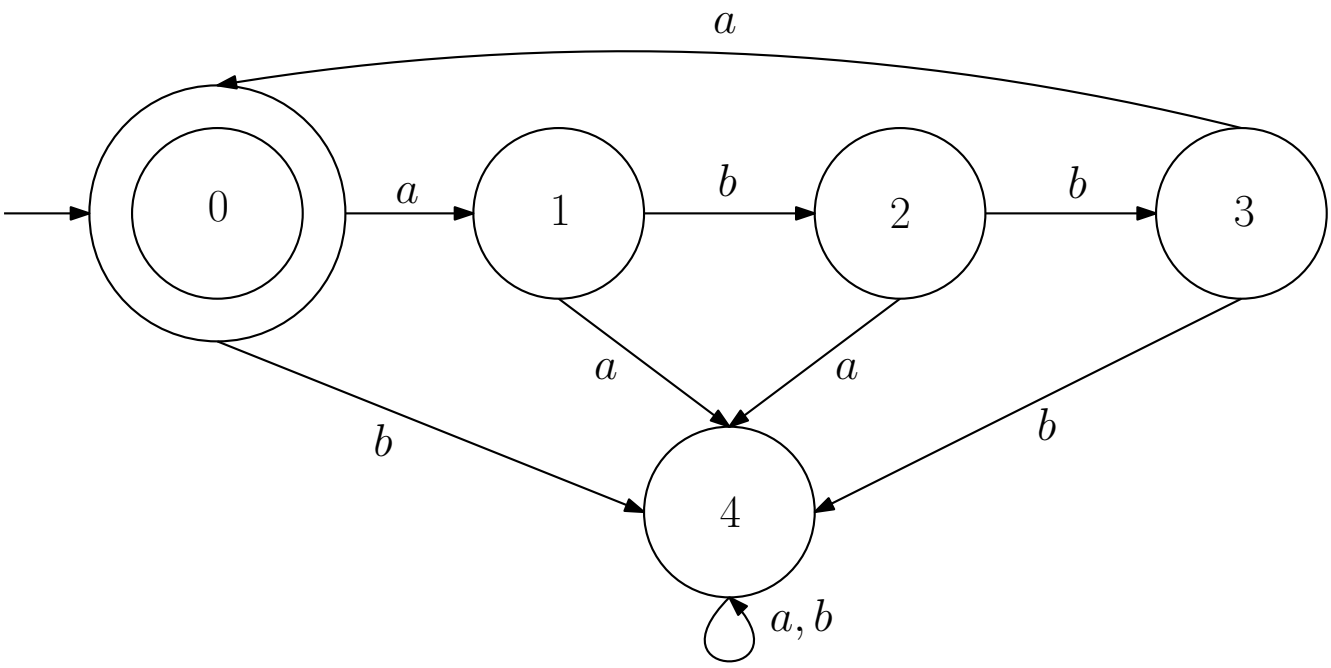
Questions from Lesson

1. Consider the following DFA:



Describe the strings accepted by this machine.

2. Build a DFA that accepts exactly the strings starting *and* ending with b.

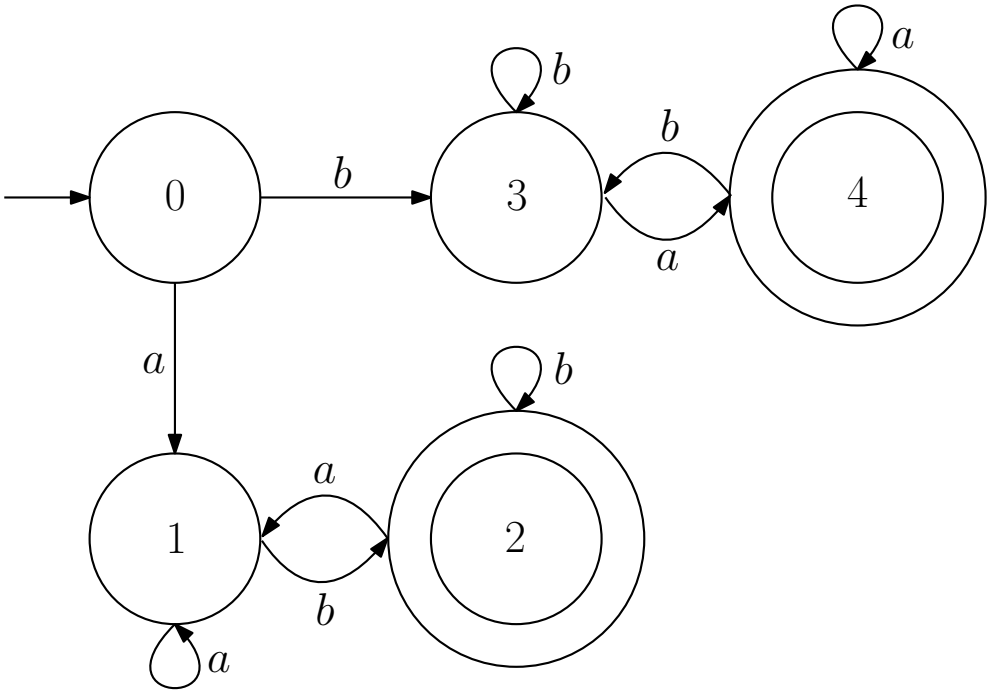
3. Build a DFA that accepts exactly the strings with baa inside them. (So $ababaa$ and $bbaabbb$ work, but $baba$ doesn't).

4. Build a DFA that accepts the strings with *exactly* two as and more than two bs.

5. Build a DFA that accepts only strings where every run of as has length 2 or 3 (so the as always come 2 in a row or 3 in a row).

Extra Questions

6. Can you describe the language accepted by the following DFA, in simple terms?



7. Consider a language built up from the following rules:

   (a) The empty string is in the language.

   (b) If you know a string is in the language, attaching *abba* or *baa* to the end of the string gives you another string in the language.

   So *abba* and *baa* are in the language, and so are *abbaabba*, *abbabaa*, *baabaa*, and so on. The short way mathematicians use to describe this language is $\{abba, baa\}^*$.

   Can you build a DFA accepting this language?

8. Have you ever tried to create a password for a website and been forced to include a bunch of special characters and obey a bunch of other rules? These conditions can be modelled by a DFA. Let's take a look at a simplified version of the situation.

   Suppose users must create a password out of the symbols $a, b, c, 0$, subject to the following conditions:

   (a) The password must contain at least one 0, and at least one letter (one of a, b, c).

   (b) The password can never contain *cab* inside it.

   Can you construct a DFA that accepts the legal passwords, and rejects the illegal ones? (Don't worry if the picture gets messy).