Math Circles – Finite Automata
Question Sheet 2

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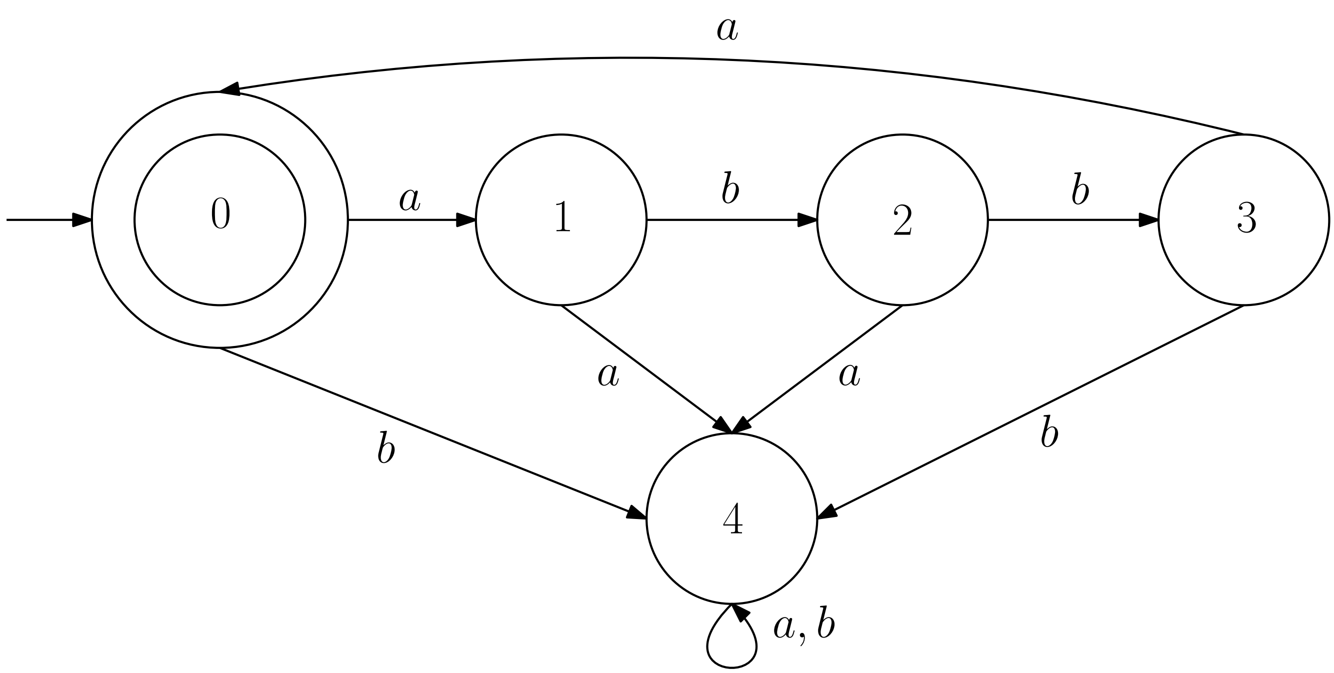
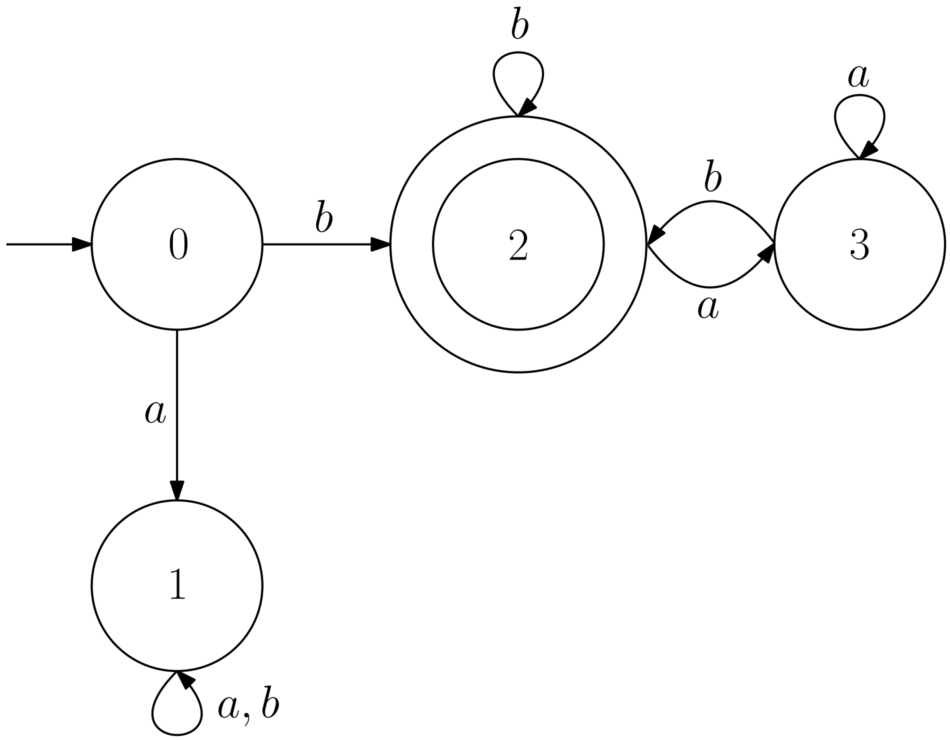
Questions from Lesson

1. Consider the DFA from last time, accepting the *abba* language:

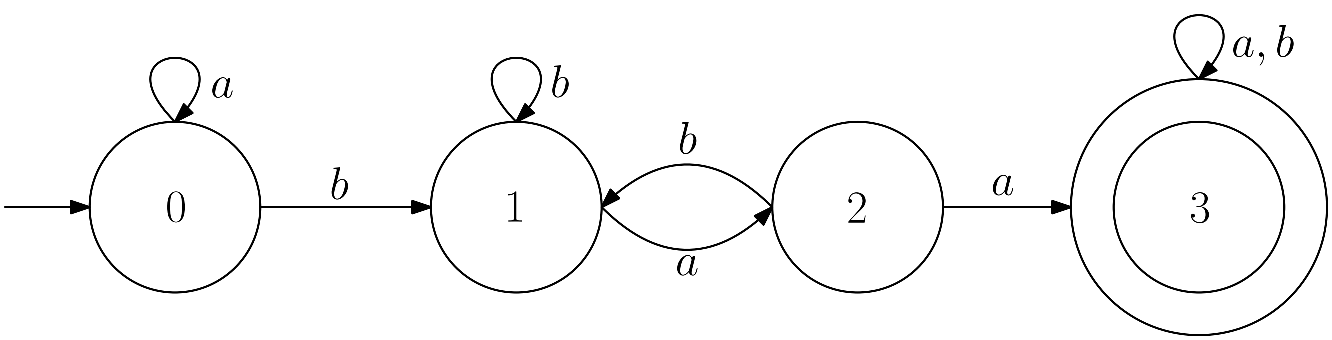


Write down a DFA accepting the *complement* of this language (the strings NOT accepted by the original DFA).

2. Here is a DFA accepting the strings that start and end with b:



Here is a DFA accepting the strings containing baa inside them:



(a) Can you build a DFA accepting the *intersection* of these two languages (the strings accepted by *both* DFAs)?

(b) Can you build a DFA accepting the *union* of these two languages (the strings accepted by *either* DFA)?

3. Is every language a regular language? If not, can you provide an example of a language that is not regular?

Extra Questions

4. The language of *legal bracketings* is a collection of strings using the letters a (left bracket) and b (right bracket) following the rule that every b in the string must have a matching a coming before it.

   Which of the following strings belong to this language?

   (a) a

   (b) ba

   (c) $abab$

   (d) $abba$

   (e) $aabb$

   (f) $ababb$

   (g) $abaababb$

   Is the language of legal bracketings regular? Why or why not?

5. The steps we talked about for building a DFA accepting the union or intersection of two regular languages always work, but sometimes it creates more states than we really need. When we built a DFA accepting the strings that start and end with b, and contain baa somewhere inside, the resulting DFA had nine states.

   Can you write down a DFA accepting the same language, but with only six states?

6. Suppose we have two regular languages, which we'll call L_1 and L_2. Consider the language of all strings belonging to L_1 but not L_2. Is this language always regular?

   If so, describe a process for building a DFA accepting this language, given DFAs accepting L_1 and L_2.

   If not, explain why not.