Grade 6 Math Circles
Fall 2018 - November 27/28

*Graph Interpretation*

Today we will be interpreting graphs. We will look at the information they represent and any patterns they have. We will also learn how people manipulate data to create graphs that mislead their audience. Finally, we will discuss Google Trends - a website where we can access statistics about the popularity of specific google searches.

**Plotting Positive Coordinate Pairs**

A coordinate pair is a pair of numbers that describe a point where a vertical and horizontal line meet on a coordinate grid. When plotting positive points, the first number describes the distance to the right of the origin. The second number describes the distance up from the origin. For example, to plot the point (5,3), you first move 5 units to the right along the horizontal axis and then 3 units up.

**Plotting Points on the Cartesian Plane**

Before we begin looking at graphs we need to understand how to plot points in the Cartesian plane. The Cartesian plane is formed by 4 quadrants. These four quadrants are separated by two axes, the x axis (horizontal) and the y axis (vertical).

In the Cartesian plane we can plot points that have negative values. Say we have a point \( P = (x, y) \):
- If $x > 0$ and $y > 0$, then $P$ is in the __________ quadrant
- If $x > 0$ and $y < 0$, then $P$ is in the __________ quadrant
- If $x < 0$ and $y > 0$, then $P$ is in the __________ quadrant
- If $x < 0$ and $y < 0$, then $P$ is in the __________ quadrant

Plotting points with negative values is simple. You still travel horizontally along the x axis before traveling vertically on the y axis to find your point, but if the point $P = (x, y)$ has a negative value for $x$ then you move left instead of right. If $y$ has a negative value then you move down instead of up.

**Example 1:** Plot and label the following points on a Cartesian plane using the grid below.

(a) $A = (-6,3)$
(b) $B = (4,-1)$
(c) $C = (-2,-3)$
(d) $D = (3,2)$
(e) $E = (0,-6)$
Common Patterns in Graphs

Linear

A graph that is linear has data points that when connected, form a line. The most basic line that can be drawn has points which are represented by the equation \( y = x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>-2</td>
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<tr>
<td>-1</td>
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<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

State whether this linear graph increases or decreases. The graph increases.

Is there a maximum value. If so, what is it? There is no maximum.

Is there a minimum value. If so, what is it? There is no minimum.

Exponential

Given the following points, plot an exponential graph on the grid below: (-2,0.25), (-1,0.5), (0,1), (1,2), (2,4), (3,8)

Special features of exponential graphs:

- approaches the x axis but never touches it
- doesn’t increase constantly (increases rapidly)

State whether this graph increases or decreases. The graph increases.

Is there a maximum value. If so, what is it? There is no maximum.

Is there a minimum value. If so, what is it? There is no minimum.
Trigonometric Sine

Given the following points, plot a trigonometric sine graph on the grid below:
(-6.28, 0), (-4.71,1), (-3.14,0), (-1.57,-1), (0,0), (1.57,1), (3.14,0), (4.71,-1), (6.28,0)

- has a wave pattern that repeats itself
- y values range from -1 to 1

State whether this graph increases or decreases. The graph both increases and decreases.

Is there a maximum value. If so, what is it? The maximum value is 1.

Is there a minimum value. If so, what is it? The minimum value is -1.

**Thinking Question**

What does it mean when we say a graph is periodic?

**Example 2:** These three patterns are common patterns because there are many real-life situations or phenomena that linear graphs, exponential graphs, and trigonometric sine graphs can be used to represent. In the table below, brainstorm some possible phenomena each graph could represent.

<table>
<thead>
<tr>
<th>Linear</th>
<th>Exponential</th>
<th>Trig Sine</th>
</tr>
</thead>
</table>

Sometimes graphs have harder patterns to analyze. Analyze this graph. Can you find any patterns? **Hint:** Look at how and when the graph increases and decreases.

Both the hare population and lynx population show a consistent trend of increase and decrease. Looking closer we see that when the hare population increases, the lynx population increases shortly after and when the lynx population increases, the hare population begins to decrease. When the hare population is smaller, the lynx population decreases. Soon after the lynx population starts to decrease, the hare population starts to increase. After this the cycle starts again.

What year had the most hares? What year had the most lynxes? 1866 was the year with the most hares. 1885 was the year with the most lynxes.

What words describe the type of relationship the Canada Lynx and Varying Hare have? The Canada Lynx and Varying Hare have a predator-prey relationship.
Graph 2

This graph shows the use of water in Edmonton on two days in February 2010. The units on the vertical axis are ML/min.

What is different about the way water was used on February 27 and February 28? The water use was much more varied on Feb. 28 than on Feb. 27.

At 14:00 on Feb. 28, what is the water demand approximately? At 14:45 the water demand is approximately 448 ML/min.

What might be the reason that the use of water was so unusual on February 28, 2010? On Sunday, Feb 28, 2010, Canada won a gold medal in Olympic men’s hockey. Epcor, Edmonton’s water provider, noticed a significant change in water use patterns during the game than on a typical weekend. The graph correlates the water use patterns to the key points of the game (i.e. the end of the first period, etc).

In the graph, the vertical axis begins at 300 ML/min, rather than 0 ML/min. How do you think the look of the graph would change if its scale went from 0 to 500 ML/min instead of starting from 300 ML/min? Why do you think it was presented this way? If the vertical axis ranged from 0 to 500 ML/min, the ups and downs of the graph would not look as extreme, since they would only span one quarter of the vertical space of the graph. By showing a vertical range from 300 to 500 ML, the graph exaggerates the variations in water use during the hockey game. This technique of “selective scale” is often used with graphs in the media, in order to focus on trends and changes.
Misleading (“Bad”) Graphs

Below are some of the ways that writers manipulate data to create misleading graphs.

- Usually the baseline (starting value on the vertical axis which is the base for measurement) is 0. Sometimes data is skewed because they use a different number. Graphs without baselines are called “truncated graphs”.

- Expanding or compressing the scale on a graph makes changes in the information seem less or more significant than they actually are.

- Sometimes writers only include certain data points and exclude other data points in order to give a false impression of the data.

- Writers sometimes choose the wrong graph on purpose in order to skew the data. For example, pie charts being used to compare the difference between groups when traditionally pie charts are used to compare parts of a whole.

- Over time, some standards for how data is visualized have been developed. By doing the opposite of a standard, graphs become confusing and even misleading to readers.

**Thinking Question**

What are some other characteristics of “bad” graphs?
Google Trends

Google trends is a website that contains information about terms being searched on the Google search engine. All you have to do is type in a word or two and it will give you the statistics about how much that term has been searched. This information is presented in a graph. These statistics can be restricted by country and by the time period. Here are a couple examples.

**Example A:** Last Friday was Black Friday (the Friday following Thanksgiving Day in the United States) which is a major shopping day for the United States and other countries (like Canada). For many it marks the unofficial beginning of the Christmas shopping season. If we look at the popularity of the term “black friday” in the United States for the past 12 months, we get a graph that looks like this

![Graph showing the popularity of black friday in the United States over the past 12 months.]

But if we change the timeline to the past five years, we get

![Graph showing the popularity of black friday in the United States over the past five years.]

This shows us that there is a pattern of people searching “black friday” every year near American Thanksgiving.

**Example B:** Google Trends can also show the rise and fall in popularity of certain things. If we look worldwide results for “fidget spinner” in the past two years we can see the rise and fall of fidget spinners which happened between March 2017 and August 2017.
Problem Set

1. State which quadrant the point is in.

   (a) $A = (6, -7)$  
   (b) $B = (-10, -11)$  
   (c) $C = (1, 4)$  
   (d) $D = (-0.25, -0.5)$  
   (e) $E = (-8, 3)$

2. Graph the points from question one on a Cartesian plane.
3. **Surge Graph**

Given the following points, plot a surge graph on the grid below: (0,0), (0.36,5), (1,7.36), (2,5.41), (4,1.46), (6,0.3), (7,0.13), (10,0.009). Make note of any special features that a surge graph has.

Special features of surge graphs:

(a) Is there a maximum value? If so, what is it?

(b) What are some real-life situations or phenomena that a surge graph could be used to represent?

4. **Extrapolation** is estimating or concluding something by assuming that existing trends or patterns will continue. In mathematics, we do this by extending a graph (with a dashed line) past the data we already know. How much water is there at six minutes?
5. In class, we talked a lot about the characteristics of misleading (or “bad”) graphs. What characteristics does a “good” graph have? That is, what about that graph makes it “good” or unbiased?

6. Do you think that misleading (“bad”) graphs should never be used? Is there a certain time or place when their use might be acceptable? Why or why not? (Note: This is a question based on opinion and thus there is no right answer)

7. Choose three terms that you think will show an interesting graph on Google Trends for Canada in the past 12 months.

   (a) What do you think each graph will look like? Why?

   (b) Use a computer and go to https://trends.google.com/trends/?geo=CA to see the actual graphs. Are they like you expected them to be? Why or why not?

   (c) If you ran these terms on the United States instead of Canada do you expect the graph to look different? Why or why not?