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Grade 6 Math Circles

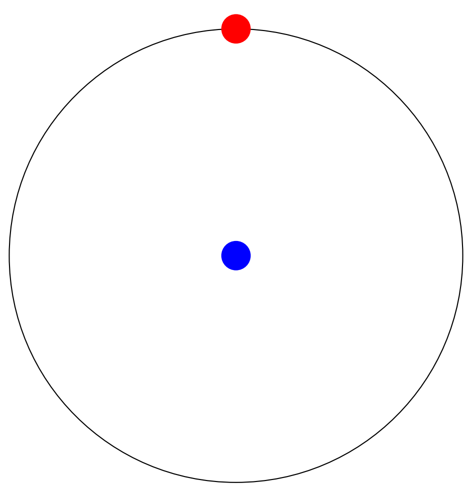
October 9 & 10 2018

Visual Vectors

Introduction

What is a **vector**? How does it differ from the numbers you already know and understand? Let's explore.

In the picture below, the outline of the circle is always 3 cm away from the blue dot in the middle (in other words, the *radius* of the circle is 3 cm). If you ask me to draw a red dot 3 cm from the blue one, that means I could draw it anywhere on this circle. What if you wanted me to draw the red dot in a particular place on the circle? How would you tell me to do that? Would I know where to draw it if you just said "draw a red dot 3 cm from the center of the blue dot"?



For me to know exactly where you want the red dot to go, you would have to tell me a direction from the blue dot to put it in. Telling me 3 cm above, or North, or 3 cm away at 90° from the horizontal, all tell me to exactly where to draw the dot.

Scalars vs. Vectors: the difference of direction

Telling me to go 3 cm from the blue dot is giving me a ***scalar***: it only tells you a size, or how much there is of something. In this case, the "how much" is a distance: 3 cm of distance. If you tell me you're moving at 20 km/h, you're still giving me a scalar: you're only telling me "how much" speed you have, and nothing else. All the numbers that you're used to seeing are scalars.

A ***vector***, on the other hand, tells you two things instead of one: it still tells you "how much", but it also tells you "in which direction". If you tell me to draw a red dot 3 cm from the blue dot, you're only giving me a scalar, without a direction. Telling me to draw the red dot 3 cm [up the page from the blue dot] is giving me a vector. With a vector, I also know which direction to go. The direction part of a vector usually comes after the number and its unit, inside square brakets like this: [ ]

As another example: telling me that you're moving at 20 km/h is a scalar. Telling me that you're moving at 20 km/h [North] is a vector. With the vector, you tell me how much speed you have, and which direction that *speed is going.*

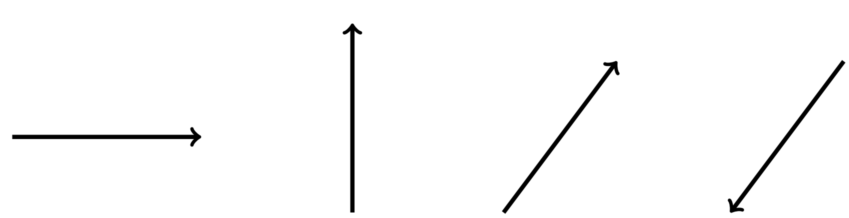
Drawing Vectors

Arrows also have a size and direction like vectors do. This is what we'll use to draw vectors:

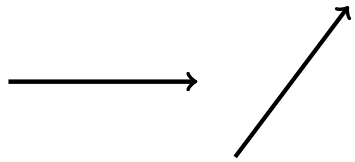
Size is represented by how long the arrow is.

Direction is represented by which way the arrow is pointing.

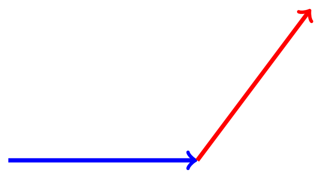
Practice: Measure these vectors with a ruler, and write out their size and which direction they are pointing in.



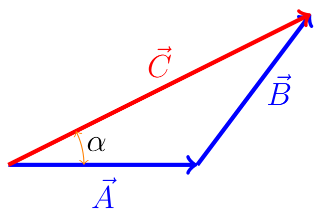
Vector Operations: addition



Adding vectors together is as simple as following the arrows. If I want to add these two vectors together, then I need to follow one, and then the other.



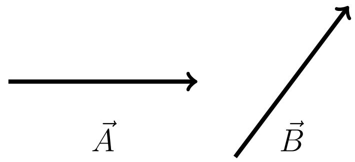
The result looks like this:



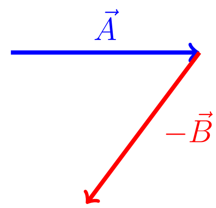
Note: We draw a little arrow on top of letters to know that they represent vectors

Practice: What is the final vector $\vec{C}$? Give your answer in centimeters, and use the angle α to give a direction. Your direction should say something like "______ degrees above/below A"

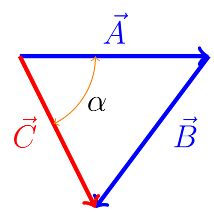
Vector Operations: subtraction



To subtract vectors $\vec{A} - \vec{B}$ we use a similar strategy. First, we follow vector $\vec{A}$, then go backwards along $\vec{B}$, by turning it around



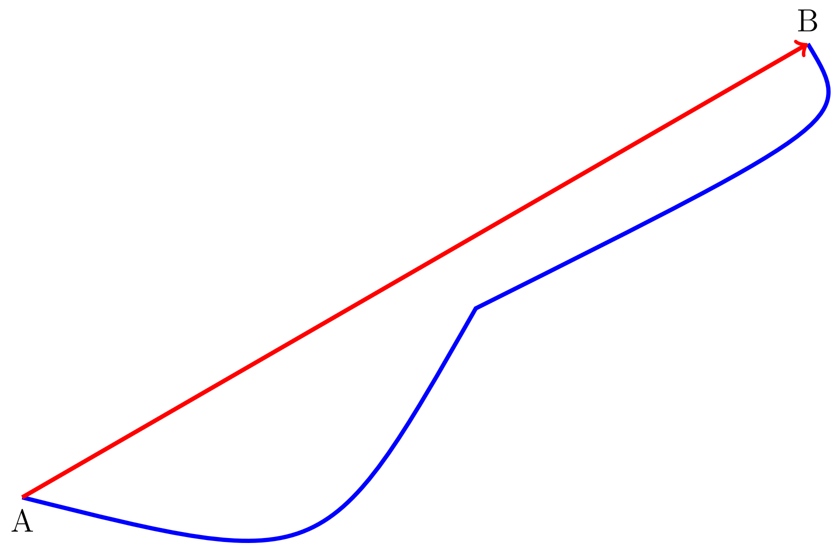
The result looks like this:



Note: We draw a little arrow on top of letters to know that they represent vectors

Practice: What is the final vector $\vec{C}$? Give your answer in centimeters, and use the angle α to give a direction. Your direction should say something like "______ degrees above/below A"

Application: distance vs displacement



Distance How long was the path you took?

Displacement How far are you from where you started?

Distance is the *scalar* that tells you "how much length there is". Displacement is the *vector* that tells you "how much length there is, and what direction it is in".

If you imagine walking from point A to point B in the diagram above, then the distance you've walked changes depending on the path you take. Walking along the blue path would take longer, and you'll have walked a greater distance.

No matter which path you take though, your displacement will NOT change. No matter what path you draw or decide to take, it doesn't change the fact that point B is "12 cm [30° up from the right]" from point A.

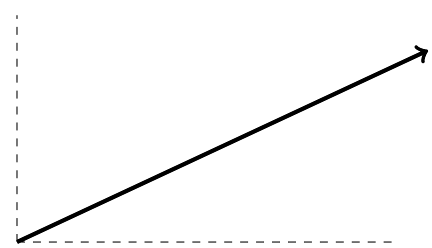
For distance and displacement to be the SAME, you would need to take the shortest, straightest path.

Exercise: Draw small displacement vector arrows along the blue path and add all the vectors together. Do you see that you will always get the red vector as your final displacement, no matter which path you take?

Problems

REVIEW

1. What is the difference between a scalar and a vector? What two things will a vector tell you? Give an example of each.
2. For each of the following, state whether the quantity given is a scalar or a vector.
   (a) Jason moved his desk 5 m.
   (b) Cindy is driving her car at 50 km/ h [North].
   (c) A ball is falling down at 3 m/s.
   (d) Alice is hiking through a forest. She's going directly South-East.
3. How would you describe the direction of the following vector on a page? How would you describe it in terms of compass directions? You do not need to measure the size of this vector, but do give an angle. (note: try to give two descriptions using compass directions)



APPLY

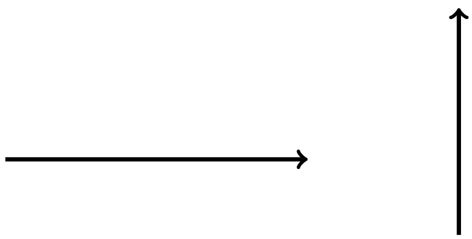
4. For each of the following, draw the vector being described.
   (a) 5 cm [60° below left]
   (b) A car is driving along the highway at 100 km/h [East]
   (note: you can draw it so that 1 cm = 10 km/h)
   (c) A boat is floating down a river at 50 km/h [40° East of South]

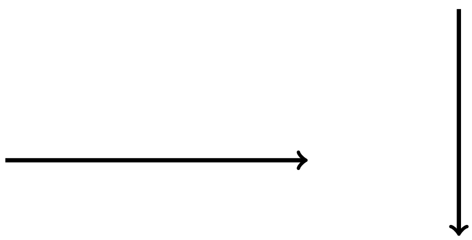
5. Add these vectors together. What is the resulting vector? Measure the size and write the direction.

   (a) 

   (b) 

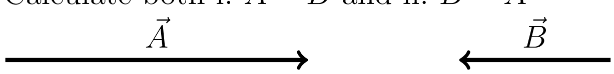
6. Add these vectors together. What is the resulting vector? Measure the size and write the direction as [degrees above or below the left or right].

   (a) 

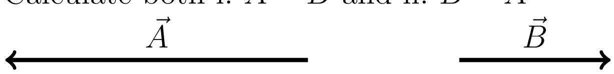
   (b) 

7. Subtract these vectors. What is the resulting vector? Measure the size and write the direction.

   (a) Calculate both i. $\vec{A} - \vec{B}$ and ii. $\vec{B} - \vec{A}$

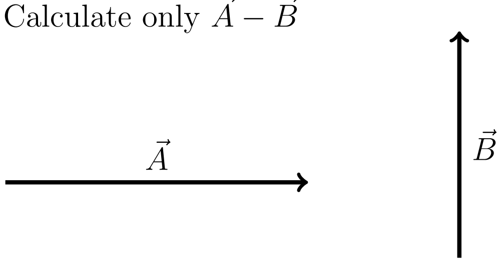
   

   (b) Calculate both i. $\vec{A} - \vec{B}$ and ii. $\vec{B} - \vec{A}$

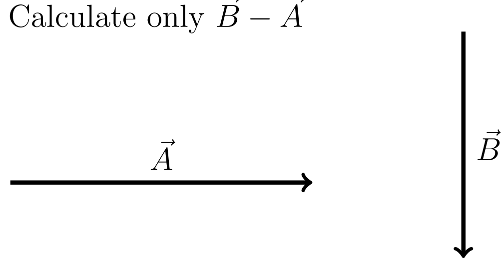
   

8. Subtract these vectors. What is the resulting vector? Measure the size and write the direction as [degrees above or below the left or right].

   (a) Calculate only $\vec{A} - \vec{B}$

(b) Calculate only $\vec{B} - \vec{A}$



9. Answer the following questions, doing the vector additions/subtractions you need.

   (a) Carl is out for a walk. He first goes 5 km [North-East], and then walks 12 km [North-West]. What is his final displacement? What is the distance he's travelled?

   (b) Bob is out for a walk. He first goes 2 km [East], and then walks 4 km [60° North from West]. What is his final displacement? What is the distance he's travelled?

   (c) Alice goes to the super market 5 km North of her house and back. Assuming she took a straight path, what is the distance she travelled? What is her displacement?

CHALLENGE

10. In question 3 you drew vectors with units of *speed* (ex. 100 km/h [East]). These are call ***velocity*** vectors. The size of a velocity vector tells you the speed something is moving at, while it's direction tells you the direction it is going. Velocity vectors can be added and subtracted just like displacement vectors can, but will result in another velocity vector.

    (a) A plane is flying through the sky. Without any wind it would be flying at 250 km/h [30° West of North]. There is a wind pushing it as well, blowing 50 km/h [North]. What is the plane's final velocity?

    (b) Tom wants to row across a river. He can row at 5 km/h. The river has a current of 3 km/h [North]. If he wants to end up directly East across the river from where he started, what direction does he need to row in? What would his total velocity be?

    (c) Tom is back at the river, but the wind is strong today. The river has a current of 12 km/h [North], so to get across, Tom decides to use a motor boat. He again wants to end up directly East across the river from where he started. If his total velocity going across the river is 5 km/h [East], what is the speed of the motor boat?