UNIVERSITY OF WATERLOO
Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles

November 28/29/30, 2017

Math Jeopardy

Introduction

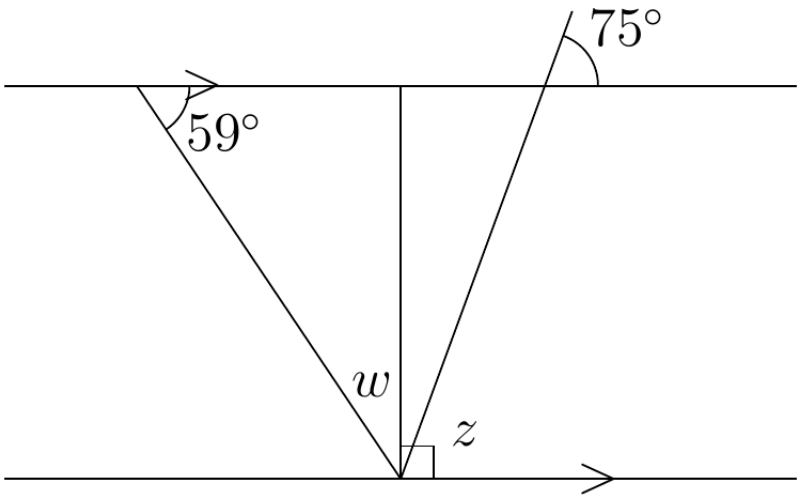
Questions will vary in difficulty with $100 questions tending to be the easiest, and $500 questions tending to be the hardest. Do your best, good luck and have fun!

Angles and Light

$100 What term describes the bending of light as it crosses the boundary separating 2 media?
Refraction

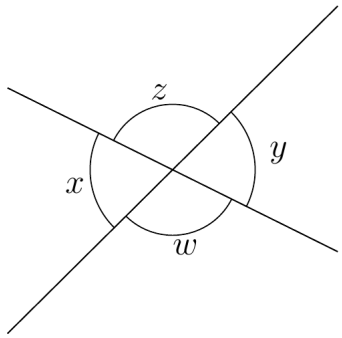
$200 What are the angle theorems we learned that you can use when you have parallel lines?
Alternate angle theorem, Corresponding angle theorem, Co-interior angle theorem.

$300 Find the missing angle:



$z = 75°, w = 31°$

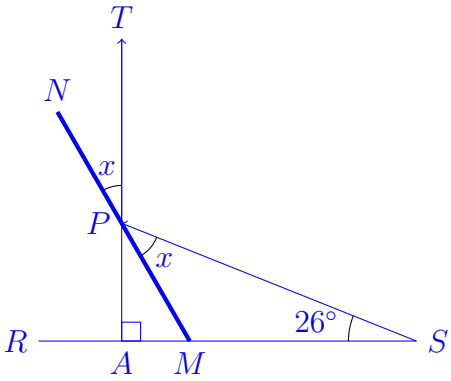
$400 Prove the Opposite angle theorem.



You can use the supplementary angle theorem twice:

$$z = 180° - y$$
$$w = 180° - y$$
$$z = w$$

$500 A beam of light shines from point S, reflects off a mirror MN at point P, and reaches point T so that PT is perpendicular to RS. What is the measure of $\angle SPM$? *(Problems, Problems, Problems, Volume 7: page 37, question 10)* **Hint:** Think about the Law of Reflection



In the diagram, extend TP to meet RS at A. Since $AT \perp RS$, then $\angle SPA = 180° - 90° - 26° = 64°$. Then by the Law of Reflection, $\angle TPN = \angle SPM$. Since, $\angle TPN$ and $\angle MPA$ are opposite angles, they are equal, so $\angle MPA = x$. Then $\angle SPA = 2 \times x = 64°$. Dividing both sides by 2 gives $x = 32°$. Therefore, $\angle SPM$ is $32°$.

Matrices

$100 What is the transpose of this matrix:

$$A = \begin{bmatrix} 13 & 2 & 1 \\ 5 & 3 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 13 & 5 \\ 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$200 Do the following matrix addition:

$$\begin{bmatrix} 4 & -2 \\ 24 & 0 \\ 7 & 30 \end{bmatrix} + \begin{bmatrix} 9 & 11 \\ 5 & 16 \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 13 & 9 \\ 29 & 16 \\ 15 & 26 \end{bmatrix}$$

$300 Do the following matrix subtraction:

$$3\begin{bmatrix} 2 & 4 & 1 \\ 12 & 0 & 5 \\ 20 & 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 6 & 1 \\ 17 & 4 & 8 \\ 53 & 5 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 2 \\ 19 & -4 & 7 \\ 7 & 4 & 2 \end{bmatrix}$$

$400 Which of these matrices could you multiply, and what would be the dimension of the final matrix?

(a) $(n \times n) \times (m \times m)$
(b) $(n \times m) \times (m \times t)$
(c) $(n \times m) \times (t \times m)$

You can multiply matrices with dimensions given by (b). The final matrix would have dimensions $(n \times t)$.

$500 Do this matrix multiplication:

$$\begin{bmatrix} 3 & 2 & 1 \\ 5 & 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & 1 \\ 2 & 5 \\ 10 & 4 \end{bmatrix} = \begin{bmatrix} 3(7)+2(2)+1(10) & 3(1)+2(5)+1(4) \\ 5(7)+3(2)+4(10) & 5(1)+3(5)+4(4) \end{bmatrix} = \begin{bmatrix} 35 & 17 \\ 81 & 36 \end{bmatrix}$$

Boolean Logic

$100 Name these logical operators:

$$\neg, \vee, \wedge, \uparrow, \downarrow$$

In order they are: NOT, OR, AND, NAND, NOR

$200 What operator this truth table for?

A	B	
True	True	True
True	False	False
False	True	False
False	False	True

A XNOR B

$300 Convert the binary number 100101 to decimal form
37

$400 Why are truth table useful?
There are a few acceptable answers for this question. The ones we were looking for here were:

(1) They give us a way to do simpler proofs.

(2) They allow us to see all possible logical outcomes easily.

(3) You can easily be certain about your results without getting lost in the logic.

$500 Evaluate this logical statement:

$$(\neg(True \wedge \neg False) \vee False) XNOR ((True \downarrow \neg False) \wedge \neg False)$$

The statement is $False$

Physics and Special Relativity

$100 What are the two Newtons Laws of Motion that we talked about?

- Newtons 1st Law: The law of inertia. Object tend to keep their velocity the same.
- Newtons 2nd Law: The force law. $\vec{F} = m\vec{a}$. Force = mass x acceleration.

$200 What kinds of frames does Special Relativity deal with? What does that mean?
Special Relativity deals with inertial frames of reference. These are frames of reference that are moving with constant velocity, with respect to any other inertial frame.

$300 What is the total force on an object if it has mass 5 kg and is moving with velocity 2 m/s [Up]?
$\vec{F} = m\vec{a}$. In this case $\vec{a} = \vec{0}$, so $\vec{F} = \vec{0}$

$400 Explain the classical Principle of Relativity, and how Einstein changed it for Special Relativity.
The classical Principle of Relativity says mechanics doesnt care about your frame, it only cares if your frame is inertial. Einsteins change was to extend this to all of nature.

$500 How much time does Alice observe passes for Bob if 10 seconds pass for Alice, and Bob flies by Alice at a constant velocity, with a speed of 0.6c
Using the time dilation equation:

$$t_B = \sqrt{1 - \frac{v^2}{c^2}} t_A$$

Where $t_A = 10$ seconds, and $v = 0.6c$ we get:

$$\begin{aligned} t_B &= \sqrt{1 - \frac{(0.6c)^2}{c^2}} \times 10 \text{ seconds} \\ &= \sqrt{1 - 0.36\frac{c^2}{c^2}} \times 10 \text{ seconds} \\ &= \sqrt{1 - 0.36} \times 10 \text{ seconds} \\ &= \sqrt{0.64} \times 10 \text{ seconds} \\ &= 0.8 \times 10 \text{ seconds} \\ t_B &= 8 \text{ seconds} \end{aligned}$$

If 10 seconds pass for Alice, then from her frame of reference she will see that only 8 seconds have passed for Bob. From Bob's frame of reference, Alice is moving at $0.6c$, just in the opposite direction. So, Bob will observe that after 10 seconds pass for him, only 8 seconds have passed for Alice.

Miscellaneous

$100 What room are you taught in?
Either DC 1304, MC 4040, or MC 4041 depending on the section.

$200 How did George Boole die?
He died of pneumonia after walking 3 miles to his University in the rain, then teaching in wet clothes, then being drenched in water by his wife in bed in an effort to cure him.

$300 How old are your Math Circles teachers?
Sydney is 19 years old. Hussam is 21 years old.

$400 How much wood would a woodchuck chuck if a woodchuck could chuck wood?
A woodchuck would chuck how much wood a woodchuck could chuck if a woodchuck could chuck wood.

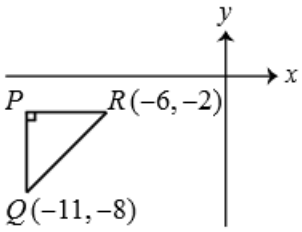
$500 What is the capital of Bulgaria?
Sofia.

Gauss Contest

$100 In $\triangle PQR$ shown, side PR is horizontal and side PQ is vertical. The coordinates of P are

(A) $(-8,-2)$ **(B)** $(-6,-8)$ **(C)** $(-11,-6)$

(D) $(-11,-2)$ **(E)** $(-8,-6)$



ANSWER: (C), The x coordinate of Q, and the y coordinate of R. *From Grade 8Gauss Contest 2016, Question 6: www. cemc. uwaterloo. ca/ contests/ past_ contests. html*

$200 The two scales shown are balanced. Which of the following is not true?

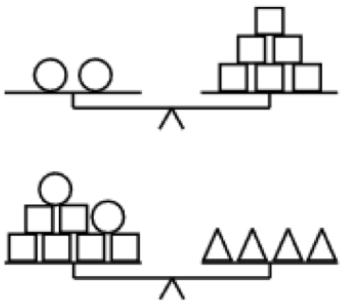
(A) ○ = △

(B) △△ = ○□□□

(C) ○ = □□□

(D) ○△ = □□□□

(E) △ = □□□



ANSWER: (D), Circle = Triangle = 3 Squares, so D is wrong. *From Grade 8Gauss Contest 2016, Question 19: www. cemc. uwaterloo. ca/ contests/ past_ contests. html*

$300 The value of $\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}$ is

(A) $\frac{3}{5}$ **(B)** $\frac{5}{3}$ **(C)** $\frac{1}{3}$ **(D)** 3 **(E)** $\frac{3}{2}$

ANSWER: (A), just need to do the calculation. *From Grade 8 Gauss Contest 2003, Question 16: www. cemc. uwaterloo. ca/ contests/ past_ contests. html*

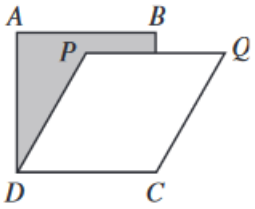
$400 How many numbers from the set $\{-5,-4,-3,-2,-1,\ 0,\ 1,\ 2,\ 3\}$ satisfy the inequality $-3x^2 < -14$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

ANSWER: (D), The numbers are -5, -4, -3, and 3. *From Grade 8 Gauss Contest 2003, Question 18: www. cemc. uwaterloo. ca/ contests/ past_ contests. html*

$500 In the diagram, $ABCD$ is a square with area $25\ \text{cm}^2$. If $PQCD$ is a rhombus with area $20\ \text{cm}^2$, the area of the shaded region, in cm^2, is

(A) 12 **(B)** 10 **(C)** 11
(D) 12.5 **(E)** 9



ANSWER: (C). See Gauss Contest solutions for details. *From Grade 8 Gauss Contest 2003, Question 24: www.cemc.uwaterloo.ca/contests/past_contests.html*

Final Jeopardy

How many pairs of positive integers (x, y) have the property that the ratio $x : 4$ equals the ratio $9 : y$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

ANSWER: (D). See Pascal Contest solutions for details. *From Grade 9 Pascal Contest 2014, Question 18:* www.cemc.uwaterloo.ca/contests/past_contests.html